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Three Essays on Equity REITs Cost of Capital

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This dissertation consists of three essays examining issues related to Equity Real Estate Investment Trusts (REITs) cost of capital. In the first essay, we examine the cost of capital characteristics of Equity REITs using firm-level data utilizing both single-factor and multi-factor cost of capital model. We find that while the evidences of time-varying risk loadings are inconclusive, the choice of estimation method in distant forecast appears to play a role. In addition, we confirm that there is a wide variation in cost of equity estimates at the firm level.

In the second essay, we model market risk loading explicit dynamic over time. We find that explicit modeling of market risk loading obtains a lower distant forecast error for about half of our sample. In addition, within the subset of firms that obtains an improved forecast, we find that a random walk market risk dynamic obtains the lowest distant forecast error.

In the third essay, we incorporate cross-sectional information to improve firm-level cost of capital estimates. Our results suggest that incorporating cross-sectional information have only a mild effect on cost of capital uncertainty. At the same time however, incorporating cross-sectional information appears to lower multi-factor cost of capital estimates and weakens Equity REITs size and book-to-market exposure at the firm level.

Three Essays on Equity REITs Cost of Capital

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To my parents, Rudolph and Astrid, and my sister, Dian,

for their endless love and support

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CHAPTER 1

On the Cost of Equity Capital of Real Estate Investment Trusts (REITs): Estimation, Stability and Predictability

1.1 Introduction

One of the fundamental concepts in financial economics is the cost of equity capital. A useful tool for financial managers who must discount future cash flows for capital budgeting decisions, it is also a critical input for analysts and investors at large in estimating the required rate of return of a risky asset. Despite the phenomenal growth of Real Estate Investment Trusts (REITs) over the last two decades, however, only a handful of papers have conducted detailed and systematic analyses of the cost of equity capital (CoEC) for Equity REITs. The objective of this study is to fill this gap in the literature. Specifically, we report findings for a comprehensive sample of 60 REITs on three related issues: (1) cost of equity capital estimates for individual REITs, and portfolios of REITs formed by property type, based on the single-factor CAPM and the three-factor Fama-French (1993) model over the sample period 1999 to 2011; (2) inter-temporal stability of factor loadings over the study period; and (3) predictability of required rates of return for both CAPM and the three-factor model.

As noted above, the extant literature on these issues is sparse and limited in scope. Three papers deserve special mention, however. These include Khoo, Hartzell, and Hoesli (KHH, 1993), Liang, McIntosh and Webb (LMW, 1995), and Chiang, Lee and Wisen (CLW, 2005)). An important point of departure of our study from the extant literature is that the analyses in previous papers are based predominantly on portfolios of REITs – both CLW and LMW follow this portfolio approach.[1] To our knowledge, only KKH present risk characteristics of individual REITs.[2] However, their analyses are limited to only 14 REITs, mainly because the study was conducted around the beginning of a new generation of REITs. We contend that developing risk measures for a comprehensive sample of REITs is a worthwhile exercise; our study provides much needed parameter estimates to analysts engaged in valuation of individual REITs. Thus, one of the main contributions of our study is the estimation of the cost of equity capital at the firm level for a comprehensive sample of modern REITs. As discussed later, the risk characteristics of REITs reveal a considerable degree of variability at the individual firm level.

A primary focus of the extant literature is the inter-temporal stability of risk attributes. This is an intuitive approach because an important consideration in using risk measures for pricing of risky assets is their stability over time. Using National Association of Real Estate Investment Trusts (NAREIT) Equity REITs index data, CLW report point estimates that suggest a decline in standard measures of risk over their sample period between 1972 and 2002. CLW estimate beta using a rolling window with 60 monthly observations. Their analyses suggest that shocks to beta have high serial correlation -- usually a symptom of a unit root process. Indeed, based on a number of tests for the presence of a unit root, against the alternative of a deterministic long-term trend, CLW cannot reject the null hypothesis of a unit root. However, the authors acknowledge

[1] CLW uses NAREIT Equity REITs index and Wilshire REIT indices. LMW uses equally-weighted portfolios of REITs.

[2] Another exception is an early unpublished study by Connors and Jackman (2000). However, in addition to the fact that the study is based on only 5 years of returns data, there are other potential limitations. We discuss this study in more detail later and for perspective, compare our findings with theirs, where appropriate.

that the correlation may be an artifact of the research design in that any two consecutive rolling estimates of beta have 59 overlapping observations.

Further evidence on the inter-temporal stability of the market model parameters is provided by CLW and LMW. Both studies rely on the *CUSUM of squares* test developed by Brown, Durbin, and Evans (1975) and both reject the hypothesis of a constant regression coefficient for the market risk factor. This inference must be viewed with caution, however, in view of the observation by Ploberger and Kramer (1990) that the *CUSUM of squares* test is appropriate primarily to examine the constancy of the residual error variance. An alternative, the *CUSUM test*, also proposed by Brown, Durbin and Evans, is widely available in standard statistical packages, and appropriate to examine the stability of beta. Unfortunately, CLW did not report results based on the CUSUM test. LMW use the CUSUM test and find that the constant beta hypothesis cannot be rejected. Finally, as one of the early studies to examine the stability of beta over the period from 1970 to 1989, KHH conclude that equity REIT betas were significantly lower in the 1980s than in the prior decade. Overall, the evidence on the inter-temporal stability of beta of equity REITs is inconclusive.

Against this backdrop of scattered evidence on the risk attributes of REITs, we first present estimates of cost of equity of individual REITs using the classical single factor CAPM of Sharpe (1964) and the three-factor model proposed by Fama and French (1993). Second, we examine the inter-temporal stability of the factor loadings from the two models; we use an informal method suggested by Fama and French (1997), and the CUSUM test developed by Brown, Durbin and Evans (1975). Third, and last, we examine the efficacy of cost of capital forecasts by CAPM and the three factor model; this analysis is useful for choosing an appropriate cost of capital model for discounting short-term vs. longer-term cash flows. Since we do not know the true process generating the risk loadings, we consider a number of possibilities including no inter-temporal variation, and time-varying such as random walk or mean reverting. If the generating process follows a random walk, rolling window regressions should give better forecasts; on the other hand, if the process is mean-reverting, then estimates based on the full sample of observations should be better. In general, for time-varying factor loadings, we expect rolling window regressions to yield more accurate of near term cost of capital estimates. These should be used for discounting near-term cash flows because the regressions are more likely to capture the current level of risk.

Our analyses start with firm-level CoEC estimates for a sample of 60 Equity REITs using monthly returns data from 1999 to 2011.[3] Over this sample period, based on the unconditional CAPM, the average CoEC estimate across all firms is 8.182% per year. This result, however, masks considerable variation both in point estimates and range at the individual firm level. For example, the annual CAPM CoEC estimate for MNRTA (Monmouth Real Estate Investment Corporation) is 5.636%, whereas that for FCH (FelCor Lodging Trust Incorporated) is 16.275%. For Fama and French (1993) three-factor model, the average unconditional CoEC across all firms is 12.638% per year. Again, we find a wide range: the annual estimate for MNRTA is 7.850% and for FCH is 26.974%. More significantly, the three-factor model consistently yields higher cost of capital estimates than CAPM; this result may be attributed to the characteristics of REITs in our sample. The data show that our sample of firms has positive exposure to the size

[3] The sample size is dictated by the number of REITs for which monthly returns data are available for the entire period 1999 to 2011. All property types are included in the sample.

factor (SMB) as well as the financial distress factor (HML) – the three-factor model prices these risks accordingly.

For comparison with previous studies and the NAREIT Index, we formed an equally-weighted portfolio consisting of all 60 firms (ALL portfolio): the annual single factor CAPM CoEC of the portfolio is 8.182% and the annual three-factor CoEC is 12.638%. As a benchmark, the CoEC of the NAREIT index over our sample period is 10.466% per year for CAPM and 16.739% per year for Fama and French (1993) three-factor model. Additionally, we grouped our firms into several portfolios based on property focus: Diversified, Health Care, Industrial, Lodging, Multi-family, Office, Retail and Self-storage. As expected, the sector-wise CoECs are clustered around the corresponding numbers for ALL portfolio. For example, Diversified portfolio is associated with CAPM CoEC of 7.120% per year and Fama and French (1993) CoEC of 11.537% per year.

Next, we focus on the inter-temporal stability of factor loadings. Previous studies suggest that for REITs, the risk loadings may be time varying. To investigate potential variation over time in factor loadings in our sample, we follow two somewhat complementary methods. One, we use an *informal test* proposed by Fama and French (1997); and two; we apply the well-known CUSUM test. We use two alternative approaches mainly because of the inconclusive findings in previous studies and the lack of power of the CUSUM test. The basic idea of the informal Fama-French "test" is that a stable beta should not deviate beyond its average standard error over time – the time series variance of beta should not be larger than the time series average of its standard error squared. These results are considered "*informal*" because of the difficulty in establishing statistical significance. The empirical results suggest that with the single factor CAPM, time variation of the market beta is large for the vast majority (58 out of 60) of firms, and cannot be explained by the size of the sampling error. Evidence from the three-factor model also suggests that the market risk loading varies with time for more than half (33 out of 60 REITs) of the sample firms. However, the loadings on SMB and HML are much less variable: only 6 out of 60 firms appear to have time-varying SMB loadings, and 17 out of 60 firms appear to have time-varying HML loadings.[4] We find similar results at the portfolio level: the market risk loading appears to be time varying while SMB and HML factors are less so. Thus, the results suggest that the SMB loading may be considered constant during our sample period.

We further investigate the null hypothesis that the factor loadings are time invariant with the CUSUM test. For the single factor model, CUSUM test rejects the null hypothesis of a constant beta for only 6 out of 60 firms; for the three-factor model, the CUSUM test rejects the null hypothesis of constant factor loadings for 4 out of 60 firms. At the portfolio level, the null hypothesis is rejected only for Health Care portfolio for CAPM. The null hypothesis is not rejected for any of the portfolios when we use the three-factor model. These results are not entirely unexpected. The CUSUM test, although used widely, has low power in detecting a large range of potential deviations from the null hypothesis (Ploberger and Kramer, 1990). On the other hand, the method proposed by Fama and French (1997) is intuitive but offers no tests for statistical significance. In sum, we deem the evidence on the inter-temporal stability of factor loadings for our sample REITs as inconclusive. Since the literature offers no compelling rationale for accepting one set of results over the other, in subsequent analyses, we explore the implications of time varying factor loadings.

[4] Following Fama and French (1993), *SMB* is the return of mimicking portfolio for the size effect; HML is the return of mimicking portfolio for the book-to-market effect.

Finally, as an important extension of the extant literature, we examine the efficacy of the models used for estimating the cost of capital.[5] The validity of the models may be judged by their accuracy in forecasting discount rates for valuation of future cash flows. Ostensibly, the ambiguity over the inter-temporal stability of factor loadings makes forecasting future discount rates a challenging task. Given the inconclusive findings from the Fama-French informal method and the CUSUM tests, we consider it prudent to examine a number of alternative scenarios such as time-invariant, random walk or mean reverting.

We also explore several estimation strategies a financial analyst might consider in selecting the appropriate cost of capital to discount future cash flows. The specific strategies utilize the full sample as well as rolling window estimates. For the random walk model, rolling regressions -- which overweight current information, may yield more accurate forecasts. The full sample of observations, on the other hand, may be more appropriate if the factor risk loading process is mean-reverting. To choose the estimation method that provides more accurate measures, we conduct out-of-sample tests of forecast accuracy.

For our 60 firms, the data indicate that for near term cash flows, there is no discernible impact on the forecast error whether we use the rolling or the full sample regressions. For distant cash flows, however, the choice of estimation method (rolling vs. full sample) has a measurable impact on the forecast errors. These results are similar whether we use the single-factor or the three-factor model, and hold also at the portfolio level.

[5] To our knowledge, the issue of forecast errors of cost of capital estimates of REITs remains unexplored. Forecast accuracy is an important question for analysts and investors.

What are the implications of our findings for valuation of REITs? One, risk factor loadings show considerable variation in point estimates and range across our sample. As such, measures based on REIT Index (*e.g.* NAREIT Index) returns – reported in previous studies - obscure the idiosyncratic characteristics of individual firms. Two, in the three-factor model, most REITs show significant sensitivity to size and book-to-market factors. As a result, CoEC estimates based on single-factor CAPM are consistently lower than those based on the Fama-French three factor model. In some respect, this result is not surprising: the majority of REITs can be classified as small to medium-capitalization firms. Moreover, as observed by several authors, book-to-market ratios may reflect the information environment of individual firms. Analysts must evaluate these two results – wide variation in cost of capital across individual REITs, and significant size and book-to-market effects -- from proper perspective when developing appropriate rates for discounting future cash flows. For relative valuation of REITs based on present values of expected future cash flows, using the single factor model may be sufficient. For more precise valuation of individual REITs, on the other hand, ignoring the sensitivity to size and book-to-market factors may be less than optimal.

Three, in-line with previous studies, there is some evidence that REITs risk factor loadings may not be inter-temporally stable. To elaborate, analyses using an informal method suggested by Fama and French (1997) reveal clear violations of the assumption of stability. The results from the CUSUM test are less convincing, however. Finally, since the tests of inter-temporal stability fail to provide any definite guidance, we investigate whether the full sample or a rolling window regression provides a better discount rate estimate for discounting future (near term, and distant) cash flows. For near term cash flows, there is consistent evidence from various tests that the estimation method has no discernible impact. Not surprisingly, however, for distant cash flows, our results suggest that the choice of the appropriate method depends on the attributes of individual REITs.

To our knowledge, this study presents the most comprehensive analyses – both in terms of sample size of REITs, and sample period – of the risk characteristics of individual REITs, as well as equally-weighted portfolios of REITs formed on property types. We contend that our findings are new and constitute a significant step towards our understanding of risk attributes at the individual firm level. However, our results leave unresolved a number of questions that future research might explore. An interesting issue is how to reconcile the significantly different costs of capital implied by the single-factor CAPM and the three-factor Fama and French (1993) model. Extant literature offers little to no guidance on which estimate one is to use in present value analyses (e.g., Fama and French (2004) and Levy (2010)). Along these lines, sources of size and book-to-market risk exposure for REITs are not well understood. A notable example, Chiang and Lee (2002) suggests that there is an upper limit to Equity REITs cash flow growth, and this exposes Equity REITs to book-to-market (value) factor risk. Another important issue to address is the inter-temporal stability of factor loadings. A full understanding of the inter-temporal properties of the underlying risk factors is a precondition for accurate cost of capital estimates. In particular, future research might focus on developing formal and powerful tests for the temporal stability of factor loadings. In this area, advances in Bayesian methods (e.g., Gelfand and Smith, 1990) offer an appropriate set of tools for both estimation and formal hypothesis tests.

The rest of the paper proceeds as follows. First, we describe the data and present a detailed discussion of cost of equity estimates for individual REITs, and portfolios of REITs by property type. These estimates are based on the CAPM and Fama-French (1993) three-factor models. We next analyze time-variation of factor loadings associated with the market risk, small firm effect, and the book-to-market effect using an *informal* method proposed by Fama-French (1997); this is followed by the CUSUM test developed by Brown, Durbin and Evans (1975). In the final part of the paper, we present an analysis of forecast accuracies using competing estimation strategies including rolling regressions, and the full sample of observations. The conclusion and a number of suggestions for future research close the paper.

1.2 Data

We obtain the sample of Equity REITs from the SNL database and National Association of Real Estate Investment Trusts (NAREIT) as of December 2011. We require that a firm has continuous monthly returns data - obtained from the Center for Research in Security Price (CRSP) dataset - from 1998 to 2011 to be included in the sample. This screen yielded a sample of 60 REITs. Our sample includes REITs owning all different property types - Diversified, Health Care, Industrial, Lodging, Multi-family, Office, Retail and Self-storage. Although we chose January 1999 as the latest start date, several REITs in the sample have complete data starting before January 1999.[6] For example, monthly return data for Duke-Weeks Realty (DRE) is available starting January 1993 while data for Boston Properties (BXP) starts from July 1997. A common sample

[6] In the results reported in the main body of the paper, we use a common period of January 1999 to December 2011 for all sample firms. We have available upon request a complete analysis where we use each firm's full sample. Our findings are not sensitive to the time period chosen for the study.

period is preferred, however, because it eliminates variation across firms due to differing sample periods. We obtain NAREIT Equity REITs index returns from that NAREIT website.

Table 1 presents the summary statistics of the 60-firm sample. For each firm, we report the ticker symbol, firm name, REIT property type and market value. Market value is measured as share price multiplied by the number of shares outstanding as of December 2011. In addition, we report the average return, standard deviation of returns, skewness, and kurtosis based on annualized 1-month holding period return. We annualized returns by multiplying monthly return by 12. Finally, we report the average and standard deviation of trading volume for each firm. The market value of the sample of REITs ranges from $13 million for Roberts Realty Investors, Inc. (RPI) to $37 billion for Simon Property Group (SPG). Firm-level returns show considerable range in both average returns and volatility; for example, RPI has an average annual return of 0.051 with a standard deviation of 1.322, while SPG has an average annual return of 0.212 with a standard deviation of 1.010. In addition, returns of the 60 firms are non-normal with more than half of the sample (34 firms out of 60) exhibiting negative skewness, and the whole sample of firms exhibiting kurtosis larger than 3. Table 1 also shows that several firms in the sample are widely traded. For instance, monthly trading volume for ProLogis Trust (PLD) is 22.73 million shares with a standard deviation of 65.24 million shares, whereas trading volume for RPI averages 0.03 million shares per month with a standard deviation of 0.05 million shares. We winsorize firm returns at 5% and 95% level to minimize the effects of outliers.

To facilitate comparison with NAREIT Equity REITs index return, we form an equally-weighted portfolio consisting of all 60 firms in our sample (ALL portfolio). We also form eight equally-weighted portfolios grouped by REIT property types. Panel A of Table 2 reports the summary statistics of our portfolios and the NAREIT index. As expected, the average annualized returns of REIT property type portfolios generally cluster around the NAREIT index's average annualized return. On average, NAREIT index returns 0.128 per year. Our sample portfolios range from a return of 0.114 per year for Diversified REIT type to 0.175 per year for Retail. Overall, the return on portfolio of 60 firms averages 0.155 per year.

We find that standard deviation of returns for the NAREIT index is lower than those of our portfolios. The annual standard deviations of our portfolios range from 0.900 for Self-storage to 1.618 for Lodging, while NAREIT index's standard deviation is 0.809 per year. Using all firms, we obtain an annual standard deviation of 1.180. Our sample includes only about half of the firms in the NAREIT index. Specifically, NAREIT Equity REITs index return in 2011 was calculated using returns data for 130 REITs. A larger number of firms constituting the NAREIT Index may contribute to NAREIT's lower standard deviation. Finally, skewness and kurtosis measures indicate that returns on all our portfolios and the NAREIT index are non-normal.

We also require returns on the market portfolio ($R_m - R_f$), and the returns on SMB and HML risk factors. The value-weighted CRSP index is used as a proxy for the market portfolio, while data on the risk-free rate, SMB and HML are obtained from Dr. Kenneth French's website. Risk premia are difficult to measure – especially using *ex-post* data. We expect that with a longer period we are better able to capture business cycles, and thus obtain more reliable estimates. Therefore, we use a long span of time, from 1927 to 2011, for all three factors (Fama and French (1997) use a similarly long period (1963 to 1994) in their study). In Panel B of Table 2, we report several statistics for the risk-free rate, the market risk premium, SMB, and HML. The yearly average, respectively, for each of these variables is: 0.036, 0.079, 0.037, and 0.047; while the corresponding standard deviation is: 0.031, 0.208, 0.142, and 0.139.

1.3 Estimating Cost of Equity Capital of REITs

The Cost of equity capital is unobservable and must be estimated with asset pricing models. In this section, we estimate CoEC using the CAPM and the Fama and French (1993) three-factor models.

1.3.1 Cost of Equity Capital based on the CAPM

We estimate the parameters of the unconditional CAPM with a time series regression of the excess return (over the risk free rate) for firm *i* on the excess return of the market portfolio:

$$R_{i,t} - R_{f,t-1} = a_i + b_i \left(R_{m,t} - R_{f,t-1} \right) + \varepsilon_{i,t} \tag{1}$$

where $R_{i,t}$ is the one month holding-period-return of firm *i*, $R_{m,t}$ is the monthly return on the market portfolio, $R_{f,t-1}$ is the yield on the one month Treasury bill, and $\varepsilon_{i,t}$ is a random shock assumed to be normally distributed with mean 0 and variance σ_i^2. Estimation is based on ordinary least squares (OLS) over the sample period from January 1999 to December 2011. To calculate CoEC using CAPM, we use the beta for each firm from equation (1), plus the risk-free rate and market risk premium from Table 2. Using a long period for factors allows us to capture business cycles and detect any discernible trends, which enhances the reliability and integrity of the estimates. Over the period spanning nearly 90 years, the average annualized risk free rate is 3.63% and the average annualized market risk premium is 7.94%.

Table 3 reports for each REIT the estimated pricing error (a_i), systematic risk (b_i), the R-squared and the CoEC based on the CAPM. The intercept is significantly different from zero for 41 out of 60 firms in the sample. To the extent that the intercept measures the goodness of fit of a regression model, the unconditional CAPM is well specified for roughly 32% of the firms in our sample. The estimation of systematic risk using OLS shows that factor loadings (b) are statistically different from zero at conventional levels for all sample firms. The average factor loading in our sample is 0.574. IRETS has the smallest factor loading of 0.222 while FCH has the largest loading of 1.594. To provide some perspective on these numbers, we note that in an early study on this topic, Connors and Jackman (2000) estimate unconditional CAPM CoEC for 49 Equity REITs using data from January 1995 to December 1999. The average factor loading reported by Connors and Jackman (2000) is 0.346, the smallest factor loading is 0.052 for Spieker Properties and the largest factor loading is 0.796 for Crescent Real Estate Equities.[7] As shown in the table, using the average annualized risk free rate is 3.63% and the average annualized market risk premium is 7.94%, the estimated average unconditional CAPM CoEC for individual firms range from 5.389% per year for IRETS to 16.275% per year for FCH. The

[7] While the unpublished study by Connors and Jackman (2000) is worthy of mention as an early attempt to measure the cost of capital of individual REITs, our findings are not directly comparable to theirs for several reasons: (1) the sample periods in the two studies are entirely different both in terms of time and length; (2) the new breed of REITs were just being formed during the period under study by Connors and Jackman, while the REITs in our sample are seasoned firms with at least 10 years of uninterrupted trading; (3) our sample period is marked by two important and significant events that could impact the risk-return trade-off of firms – the 2008 financial crisis, and consolidations in the REITs industry since 1999 (see Ambrose, Highfield and Linneman (2005)). For example, Equity Office Properties acquired Spieker Properties in 2001. Equity Office Properties in turn was acquired by Blackstone Group in 2007. Following the consolidation, firms may have different sensitivities to a broad-based market Index.

average CoEC across our 60-firm sample is 8.182% per year. Finally, we also report the unconditional R-squared in Table 3. In the aggregate, the CAPM explains 18.6% of returns in our sample, with a range of 4.6% for RPI to 38.7% for FCH.

1.3.2 Cost of Equity Capital using the three-factor model

Fama and French (1993) propose a three-factor model to explain the cross section of expected returns. We regress monthly excess returns of each firm on three factors to estimate the parameters of this model:

$$R_{i,t} - R_{f,t-1} = a_i + b_i\left(R_{m,t} - R_{f,t-1}\right) + c_i\left(SMB_t\right) + d_i\left(HML_t\right) + \omega_{i,t} \qquad (2)$$

where the first factor is the market excess return, *SMB* is the return of mimicking portfolio for the size effect, HML is the return of mimicking portfolio for the book-to-market effect, and $\omega_{i,t}$ is normally distributed with mean 0 and variance σ_i^2. Since both SMB and HML represent returns on zero investment and zero risk portfolios, risk free rate need not be subtracted from SMB and HML factors. We estimate the factor loadings using OLS and use them to compute the three-factor CoEC. For each firm, the risk premium associated with the small-firm effect is calculated as the regression coefficient on SMB in equation (2) multiplied by the average return on the SMB portfolio. The risk premium on the book-to-market factor and the market risk factor is calculated similarly.

Table 4 reports for each firm the estimated pricing error (a_i) and systematic risk factors attributed to market risk premium (b_i), size effect (c_i) and book-to-market effect (d_i). We also include the R-squared and the CoEC. As expected, adding the size and book-to-market effects to the model improves its goodness of fit. As shown in the table, the model appears to be well specified for 31 out of 60 firms in the sample. Twenty-nine out 60 firms (50%) show intercepts that are statistically different from zero. In addition, factor loadings for market risk premium and HML are statistically different from zero at 5% level for all 60 sample firms, while factor loadings for SMB are statistically different from zero for 50 out of 60 sample firms. The average factor loading for market risk premium is 0.568, the average factor loading for SMB is 0.369, and that for HML is 0.666.[8] Indeed, our results show that even at the firm level, the HML regression coefficient is higher than the SMB coefficient after the 1990s. In other words, the book-to-market effect attracts a higher premium than the small-firm effect for our sample REITs. The coefficients imply that small-firm effect has an average risk premium of 1.36% per year while book-to-market effect has an average risk premium 3.13% per year.

In corroboration of our findings, Chiang and Lee (2002) conclude that equity REITs behave like small cap stocks after 1990s with value (book-to-market) effect being stronger than the size effect.[9] Chiang and Lee (2002) argue that the growth rate of cash flows of Equity REITs may be limited. For example, leases attached to the underlying real estate may set an upper limit on the

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[8] For comparison, Connors and Jackman (2000) report an average factor loading for market risk premium of 0.547, an average factor loading for SMB of 0.407 and an average factor loading for HML of 0.576 under unconditional Fama and French (1993). The regression coefficients for HML and SMB are comparable also with those reported by CLW (2005). Using NAREIT index returns over the period January 1992 to December 2002, CLW report a higher HML factor loading of 0.523 compared to SMB factor loading of 0.328.

[9] In contrast, CLW find HML regression coefficient to be lower than SMB regression coefficient in their pre-1992 sample. Similarly, Peterson and Hsieh (1997) observe that HML effect is lower than that of SMB using portfolio level Equity REITs data from July 1976 to December 1992. Direct comparison with these results is not possible because our sample period post dates the period.

growth rate of annual rent. To the extent that the HML factor reflects future prospects, HML loading is expected to be significant for Equity REITs. As for the size effect, several past studies have documented small-firm risk factors in Equity REITs. Peterson and Hsieh (1997) documented significance of SMB using data for a sample of Equity REITs from 1976 to 1992. Our data reveal that the significance of the size factor persists following the phenomenal growth of REITs in the late 1990s. Further, Chiang and Lee (2002) reported that Equity REITs index consistently show sensitivity to value (book-to-market) effect over time. On the contrary, during some periods studied by Chiang and Lee (2002), REITs show no sensitivity to the size factor. Clearly, size and book-to-market factors affect Equity REITs differently over time.

1.3.3 Cost of Equity Capital of REIT Type portfolios and the NAREIT index

Table 5 reports the regression coefficients, R-squared and CoEC for REIT Type portfolios, a portfolio that consists of all 60 firms in the sample which we designate as the *ALL portfolio*, and the NAREIT index. Panel A presents the estimates from CAPM. CoEC of ALL portfolio (8.182% per year) is lower than that of NAREIT (10.466% per year). Market risk premium loadings are 0.574 and 0.862 for ALL portfolio and NAREIT Index, respectively, both loadings are statistically different from zero at 1% level. The difference in CoECs between NAREIT index and the ALL portfolio may be attributed to the fact that the NAREIT index includes more REITs than the ALL portfolio. Further, since our sample includes REITs with continuous operating history since 1998, the ALL portfolio is comprised of larger and more established Equity REITs. We did, however, obtain similar pricing error for ALL portfolio and the NAREIT Index. Pricing error is 0.009 for ALL and 0.007 for NAREIT - both are statistically different from zero. R-squared is also similar at 35.1% and 39.6% for ALL portfolio and NAREIT Index, respectively. As expected, the CoEC of REIT Type portfolios are spread around the corresponding values for the ALL and NAREIT portfolios. CoEC for REIT Type portfolios ranges from 6.698% per year for Self-storage to 12.108% per year for Lodging REITs. R-squared ranges from 10.7% for Self-storage to 41.9% for Lodging. Furthermore, all market risk premium loadings are statistically different from zero at 1% level.

Panel B of Table 5 reports three-factor model estimates. Annual CoEC for the ALL portfolio and NAREIT index is 12.638% and 16.739%, respectively. The model explains 59.6% of the variation in ALL portfolio, and 64.3% of variation in NAREIT index. Estimated factor loadings for the ALL portfolio are 0.568, 0.369 and 0.666 for the market risk premium, SMB and HML factors, respectively; the corresponding numbers for NAREIT are 0.874, 0.442 and 0.962, respectively. As noted before, the larger number of REITs in the NAREIT Index may contribute to the difference in loadings between ALL and NAREIT portfolios. The lower proxy for distress risk as implied in the HML loading is possibly due to the presence of larger and more established firms in the ALL portfolio. The ALL portfolio's pricing error is statistically different from zero; Fama and French (1993) three-factor model appears to fit the NAREIT data better. REIT Type CoEC ranges from 10.381% per year for Self-storage to 18.694% per year for Lodging, clustering around corresponding estimates for ALL and NAREIT portfolios.

1.3.4 Comparison of CAPM and Three-Factor Model Estimates

A comparison of the estimates from the unconditional CAPM and the three-factor Fama-French models in Tables 3 and 4 reveal several interesting patterns. First, the unconditional CAPM estimates are on average 4.46% lower than unconditional Fama-French estimates at the individual firm level. Since the SMB and HML factors are significant for most REITs in the sample, this result is intuitive. Second, Panels A and B of Table 5 indicate that CAPM and the three-factor models yield different CoEC estimates at the portfolio level.[10] Finally, NAREIT index's annual CAPM CoEC is lower than that for our 60-REIT ALL portfolio. NAREIT index consists of more firms than our sample. In addition, our sample includes firms with continuous operating history from 1998 which limits it to more seasoned firms. As expected, CoECs of REIT Type Portfolios group around the corresponding values for the ALL and NAREIT portfolios.

What do these findings imply for use of CoEC estimated by CAPM and the Fama-French three-factor model for discounting uncertain future cash flows for valuation purposes? First, cost of capital vary over wide ranges for individual REITs under both CAPM and Fama-French three factor models. Consequently, use of factor loadings derived from aggregate Index returns may be inappropriate for firm-specific valuation analysis. Second, consistent with expectation, incorporation of the SMB and HML factors improves the goodness of fit of the estimated model, and also results in cost of capital implied by CAPM to be consistently and significantly less than that implied by the three-factor model. As such, without any adjustments, using discount rates estimated by CAPM will consistently yield higher values of risky cash flows than by the three-

[10] Fama and French (1997) also report differences across the two models. Using industry level data from 1963 to 1994, Fama and French (1997) shows that CoEC minus the risk free rate under unconditional CAPM for Drugs industry is 4.71% per year, and annual 0.09% CoEC under the unconditional three-factor model. Similar differences were reported by Connors and Jackman (2000).

factor return generating model. As guidance for analysts engaged in developing cost of equity capital, we recommend two adjustments. For purposes of comparison and ranking among risky assets, the choice of the model should not affect the decision if the same model is applied consistently for all assets and across time. However, for absolute valuation of individual assets, a possible solution would be to estimate cash flows after adjustment of systematic risk for CAPM, and for the three factor loadings according to the Fama-French model, and then use the appropriate discount rate estimated by the respective model. A final alternative would be to choose the model for individual assets based on parsimony and the significance of additional risky factors due to size and book-to-market.

The above estimates of CoEC assume a constant loading over the estimation period. Previous studies, however, suggest that estimates of factor loadings may be inter-temporally unstable. Next, we investigate time variation in factor loadings of CAPM and Fama and French (1993) three-factor model for the 60 Equity REITs under consideration.

1.4 Inter-temporal Stability of Factor Loadings

In this section, we investigate potential time variation in factor loadings using two alternative approaches: an *informal* method proposed by Fama and French (1997), and a formal CUSUM test. We use these alternative approaches because previous studies, using variants of CUSUM, lead to inconclusive results. We start by exploring time variation in factor loadings using the Fama-French methodology, followed by a formal CUSUM test.

1.4.1 Time Variation in Factor Loadings as per Fama and French (1997) Method

We investigate inter-temporal stability in CAPM's beta using the *informal* method proposed by Fama and French (1997). Using a rolling window of 3 years, we obtain a time series of betas from equation (1) using OLS. Assigning β to be the true market factor loading and b its estimated value from rolling window regressions, the implied variation of the true β is given by

$$Var(\beta) = Var(b) - \overline{s^2\left((X'X)^{-1}\right)_{2,2}} \tag{3}$$

where Var(b) is the sample variance of the rolling time series estimates, and $\overline{s^2\left(\left(X'X\right)^{-1}\right)_{2,2}}$ is the sample average of the squared standard error of the time series estimates of b. In this method, $\mathrm{Var}(\beta)$ is *informally* viewed as the implied variation over time of the true factor loading β. If the right hand side of Equation (3) is positive, the implication is that the true beta is time-varying. A zero or negative value indicates a constant (non-time-varying) beta. Intuitively, a zero or negative $\mathrm{Var}(\beta)$ indicates that variation in the estimated value of β is attributable to measurement error (given by $\pm$ the standard error of the estimate), rather than a change in the true factor loading. It should be stressed, however, that the sampling distribution of $\mathrm{Var}(\beta)$ is unknown hence the statistical significance associated with $\mathrm{Var}(\beta)$ cannot be obtained. As such, we limit our interpretation and inference to the sign (positive or negative) of the difference in variances. Values of $\mathrm{Var}(\beta)$ for individual REITs, as implied by CAPM, are reported in Panel A of Table 6. As shown in the table, 58 out of 60 firms have positive $\mathrm{Var}(\beta)$, suggesting that the true market risk premium loading in the single factor model is not inter-temporally stable for nearly the whole sample of REITs.

Using similar methodology, we calculate implied variations of the true loadings for market risk, SMB and HML under a three-factor model. We obtain a time series for the market risk loading *b*, for SMB *c*, and for HML *d* using a 3-year rolling window regression over the study period. Assigning β, χ and δ to be the true factor loadings and *b*, *c* and *d* to be their estimated values, respectively, the implied variations are:

$$
\begin{aligned}
\mathrm{Var}(\beta) &= \mathrm{Var}(b) - \overline{s^2\left((X'X)^{-1}\right)_{2,2}} \\
\mathrm{Var}(\chi) &= \mathrm{Var}(c) - \overline{s^2\left((X'X)^{-1}\right)_{3,3}} \\
\mathrm{Var}(\delta) &= \mathrm{Var}(d) - \overline{s^2\left((X'X)^{-1}\right)_{4,4}}
\end{aligned}
\tag{4}
$$

Panel B of Table 6 presents the results for individual firms. Somewhat consistent with the findings of the CAPM model, 33 out of 60 firms show positive $\mathrm{Var}(\beta)$, suggesting that the market risk loading may be time-varying under the Fama-French (1993) three-factor model for a large proportion of REITs. In contrast, the true SMB and HML factor loadings show less variation over time – only 6 out of 60 firms have positive $\mathrm{Var}(\chi)$, while 17 out of 60 firms have positive $\mathrm{Var}(\delta)$. These results are consistent with the notion that variations in the estimated regression coefficients for size and book-to-market are due to measurement errors for the majority of sample firms.

Table 7 reports $Var(\beta)$, $Var(\chi)$ and $Var(\delta)$ under the CAPM and three-factor model for various portfolios and the NAREIT index. The data reveal clearly discernible patterns for the market risk loading at the portfolio level. The market risk regression coefficient varies over time also at the portfolio level. Panel A suggests that the CAPM beta is not stable for any portfolio, while Panel B suggests that for the three-factor model, the market risk loading is varying over time for the vast majority (9 out of 10) of sample portfolios. The true HML loadings also show patterns consistent with time-variation for 4 out of 10 portfolios: $Var(\delta)$ is positive for Industrial, Lodging, and Multi-family REIT Type portfolios, in addition to NAREIT index. Panel B suggests a time-invariant SMB loading, however. These patterns are consistent with the trends in the firm-level data; in general, HML loadings show a greater tendency to vary with time as compared to SMB loadings.

1.4.2 Inter-temporal Patterns of Factor Loadings as per CUSUM Test

In the previous section, the Fama and French (1997) *informal* tests reveal patterns that are consistent with lack of inter-temporal stability in the market risk factor loading under CAPM. Under the three-factor model, the *informal* method corroborates the notion that the market risk loading may be time varying for a large proportion of REITs in our sample. SMB and HML loadings, on the other hand, may be time varying for a significantly smaller number of REITs.

As mentioned above, Fama and French (1997) is not a formal test because it does not lend itself to significance testing. In this section, we use the CUSUM test as a complement to our investigation. We utilize the CUSUM test with 1%, 5% and 10% levels of significance using

Stata's *cusum6* functionality as in Baum (2000). The null hypothesis of the CUSUM test for CAPM is,

$$b_{i,1} = b_{i,2} = \ldots = b_{i,T} \tag{5}$$

where $b_{i,t}$ (for t=1, 2, . . . , T) is the market risk loading at time t for firm i as shown in equation (1). The test is based on the behavior of recursive residuals w_t, defined as,

$$w_t = \frac{y_t - x_t^{'} b_{t-1}}{\sqrt{1 + x_t^{'} \left(X_{t-1}^{'} X_{t-1} \right)^{-1} x_t}} \tag{6}$$

for $t = k+1, \ldots, T$ and where y_t is the stock excess return at time t, x_t is the excess market return in the CAPM case (k=2), and the three factor returns in the Fama-French case (k=4), X_{t-1} is a matrix of k variables from *1* to *t-1*, and b_{t-1} is the regression slope based on the first *t-1* observations. Following Brown, Durbin and Evans (1975), the CUSUM test statistic W_t is defined as,

$$W_t = \frac{1}{s} \sum_{j=k+1}^{t} w_j \tag{7}$$

where s is the sample standard deviation of the recursive residuals using the whole sample. Two significance lines define the region for the null hypothesis to be true - $\left\{ k, \pm c\sqrt{T-k} \right\}$ and $\left\{ T, \pm 3c\sqrt{T-k} \right\}$ with c equal to 0.948 for 5% significance level. If W_t crosses the boundaries of

$\left\{k, \pm 0.948\sqrt{T-k}\right\}$ and $\left\{T, \pm 3\left(0.948\right)\sqrt{T-k}\right\}$, the null hypothesis of equation (5) is rejected at 5% level. The value of c for the 1% significance level is 1.143, while that for 10% significance level is 0.850.

Panel A of Figure 1 exhibits the relevant CUSUM plots with the 1%, 5%, and 10% levels of significance. The null hypothesis (5) is rejected for 6 out of 60 firms under the CAPM model - the null is rejected at the 1% the level for LTC; rejected at the 5% level for OHI and FRT; and, rejected at the 10% level for TCO, DDR and ALX.[11] These results are inconsistent with the earlier findings of the informal Fama and French method; one may easily conclude that the evidence from the two tests is inconclusive. We note two important caveats, however. First, as some previous studies have noted, the CUSUM test has low power (see for example, Ploberger and Kramer (1990)); and second, by pure chance at the 10% level, we would expect 6 out of 60 firms in the sample to have time varying loading over the long run. From a practitioner's perspective, this may be interpreted as weak evidence for inter-temporal variation in the CAPM beta. To be conservative and err on the side of caution, in subsequent analyses, we consider both possibilities: constant and non-constant CAPM beta.

The null hypothesis for the three-factor model under the CUSUM test is:

$$\begin{aligned} b_{i,1} &= b_{i,2} = \ldots = b_{i,T} \\ c_{i,1} &= c_{i,2} = \ldots = c_{i,T} \\ d_{i,1} &= d_{i,2} = \ldots = d_{i,T} \end{aligned} \tag{8}$$

[11] The complete set of CUSUM plots for all 60 firms is available on request from the authors.

where $b_{i,t}$, $c_{i,t}$ and $d_{i,t}$ is the market risk premium, SMB and HML factor loadings for firm i at time t=1, 2, . . . , T as shown in equation (2). The null hypothesis is rejected for 4 firms out of 60 firms in the sample. The CUSUM plots for the 4 firms under the three-factor model are exhibited in Panel B of Figure 1. We reject the null hypothesis (8) at 1% level for LTC and at 5% level for OHI. At 10% level, null is rejected for firm OFC and ALX. As with the findings for the CAPM, the results of the CUSUM test under the three factor model are at variance with the informal Fama-French method. According to the Fama-French method, 33, 6 and 17 firms are identified as having time-varying market risk premium, SMB and HML loadings, respectively. Once again, the results must be considered inconclusive; thus, in subsequent analyses, we allow both constant and time-varying loadings.

Finally, we analyze the CUSUM plots for REIT Type portfolios, the ALL portfolio as well as the NAREIT Equity REIT index; to economize on space these graphs are not reported here, we just describe the results. The null hypothesis of constant market risk premium loading under CAPM is rejected at 5% level for Health Care REIT Type. Using the three-factor model, however, we cannot reject the null hypothesis (equation 8) of constant factor loadings for any of the REIT property type portfolios. Once again, this evidence is inconsistent with our previous finding from the Fama-French informal method.

1.5 Forecasting Power of the Cost of Equity Capital

Valuation of uncertain cash flows requires development of future discount rates. In practice, a manager discounting short-term cash flows will be interested in near term discount rates. A rolling regression will therefore be a likely estimation method. To discount distant cash flows, however, the suitable estimation method will depend on behavior of factor loadings. In this section, we investigate the performance of out-of-sample CoEC with loadings estimated via rolling window or full sample regressions. To incorporate time-variation, we assume that true factor loadings behave either as a random walk or a mean reverting process.

For a random walk, the best estimate of the next observation is its most recent value. Hence, a rolling regression may provide a more accurate estimate of the future risk loading because the estimate is heavily weighted with the most recent information. On the other hand, for a mean-reverting process, the long-run average represents the best unbiased forecast of the next observation. To explore both scenarios, we first compare the relative efficacy of CoEC estimated with CAPM using a full sample OLS regression, a $2^1/_2$-year rolling window, and a 4-year rolling window. We then apply the same method to the three-factor model.

To evaluate the performance of the estimation strategies, we examine the predictive ability of a particular model. One month, one year, and 5 year out-of-sample forecast accuracy of each CoEC estimate is compared. We focus on near term accuracy to discount near term cash flows, and a more distant forecast accuracy to discount longer term cash flows. For near term forecasts, it is intuitive to expect estimates based on a rolling regression to be more accurate as they reflect the current CoEC. For far-off forecasts, the precision depends on whether risk loadings follow a mean reverting process or a random walk. If factor loadings are mean reverting, a full sample regression is expected to provide a more accurate distant forecast as the longer period better captures mean reversion. On the other hand, if loadings follow a random walk, we expect a rolling regression to be more accurate even for far-off forecasts as a rolling regression provides more recent information on CoEC**.**

1.5.1 Forecasts based on the single factor CAPM

We start by examining forecast errors of CAPM estimates with rolling windows. Forecast errors are calculated as follows. For each firm i, we obtain a monthly time series of factor loadings using a rolling regression with $2^1/_2$-year and 4-year rolling windows from January 1999 to December 2011. Given these factor loadings, we obtain a time series of 1-month forecast errors:

$$\varepsilon(1)_{t+1,i} = R_{t+1,i} - \hat{R}_{t+1,i} = R_{t+1,i} - b_{t,i}(\bar{R}_m - \bar{R}_f) \tag{9}$$

where, period t to t+1 represents 1 month. $b_{t,i}$ is the factor loading at time t for firm i, $R_{t+1,i}$ is the realized annualized return for firm i at time t+1, $\bar{R}_m$ is the long run average of CRSP value-weighted return, and $\bar{R}_f$ is the long run average risk-free return. This process generates a monthly forecast error $\varepsilon(1)_{t+1,i}$ series for both $2^1/_2$-year and 4-year rolling windows. We truncate the monthly $\varepsilon(1)_{t+1,i}$ series for $2^1/_2$-year window to match the $\varepsilon(1)_{t+1,i}$ series for the 4-year rolling OLS regressions. From the 1-month forecast error series $\varepsilon(1)_{t+1,i}$, we obtain the time series of τ-year absolute forecast errors as

$$\varepsilon\, abs(\tau)_{t+1,i} = \left| \sum_{j=1}^{\tau} \varepsilon(1)_{t+1-j,\,i} \right| \tag{10}$$

where τ = 12 months for a 1-year horizon, and 60 months for a 5-year horizon. In the final step, we compute the mean and standard deviation of the time series for 1-month, 1-year and 5-year absolute forecast errors.

This methodology is appropriate if the factor generating process follows a random walk. If, however, mean reversion is a more accurate characterization of the beta generating process, then estimates based on the full sample of observations should provide better forecasts. The procedure in this case is the same as above -- except we use the full sample to obtain b_i for each firm *i*. Again, we match the forecast period to that of the rolling window regressions. Finally, the time series mean and standard deviation of the 1-month, 1-year and 5-year absolute forecast errors are easily computed.

The mean absolute forecast error for the 60 firms in the sample is reported in Table 8. At forecast horizon of 1-month, there is no perceptible difference between using $2^1/_2$-year, 4-year or the full sample to estimate CAPM CoEC. For example, the forecast error for HCN is approximately 0.57 regardless of the estimation method. Most of the sample firms show the same general pattern. Averaging over all 60 firms, the mean absolute forecast error estimated using the $2^1/_2$-year rolling window is 0.685, followed by 0.687 for a 4-year window, and 0.683 for the full sample. These results imply that to value near term cash flows with the CAPM, one can be indifferent between a rolling or full-sample estimate of the discount rate.

In calculating the appropriate discount rate for valuation of distant cash flows, however, the choice of the estimation method (rolling vs. full sample) appears to play a role. Indeed, the impact of the estimation method chosen differs across individual REITs. To illustrate, our analysis of HCN shows that the mean absolute forecast error at 1-year horizon estimated using a $2^{1}/_{2}$-year rolling window is of the order of 1.584. Estimates using a 4-year rolling and full sample OLS are associated with forecast errors of 1.660 and 1.737, respectively. On the other hand, at 5-year forecast horizon, HCN's mean absolute forecast errors estimated via $2^{1}/_{2}$-year rolling, 4-year rolling and full sample OLS are 4.212, 5.044 and 6.550, respectively. Based on these results, we can infer that for valuation of HCN's more distant cash flows, a rolling regression yields a more accurate CAPM discount rate. In contrast, HR's mean absolute forecast error at 1-year horizon is 1.999, 1.986 and 1.868 when we estimate CAPM using $2^{1}/_{2}$-year rolling, 4-year rolling and full sample OLS. At the 5-year horizon, the mean absolute forecast error is 3.108, 2.740 and 1.785 estimated with the $2^{1}/_{2}$-year rolling, 4-year rolling and full sample OLS. Consequently, to value HR's more distant cash flows, the full sample OLS will generate a more accurate CAPM discount rate. For the sample as a whole, the effect of the estimation method for distant discount rates is unclear. At 1-year forecast horizon, full sample OLS is the better method with mean absolute forecast error of 2.566. At 5-year forecast horizon however, the $2^{1}/_{2}$-year rolling window obtains a lower mean absolute forecast error at 4.518.

In Table 9, we present the standard deviations of the absolute forecast error for each firm, using CAPM as the model. At 1-month horizon, the standard deviations across estimation methods are comparable. On average, standard deviation of the absolute forecast errors is 0.484. At longer forecast horizon, the choice of estimation method has a discernible impact. For HCN, full sample OLS shows a lower dispersion at 5-year forecast horizon while $2^{1}/_{2}$-year rolling obtains a lower dispersion at 1-year forecast horizon. In comparison, for HR, full sample OLS has a lower standard deviation for both 1-year and 5-year forecast horizons. In the aggregate, using more data to obtain factor loading estimate results in a lower standard deviation of forecast error - at 1-year forecast horizon, the standard deviation is 1.768, 1.765 and 1.739 for $2^1/_2$-year rolling, 4-year rolling and full sample OLS estimates. At the longer 5-year horizon, the pattern is unclear - the standard deviation is 2.553, 2.629 and 2.439 with $2^1/_2$-year rolling, 4-year rolling and full sample OLS, respectively.

Next, we examine out-of-sample performance of our portfolios and NAREIT index. Panel A of Table 10 reports the mean absolute forecast errors of our portfolios under CAPM. The 1-month forecast horizon has similar mean absolute forecast error regardless of the estimation method used. Similar to individual firms, the estimation method appears to influence accuracy of forecast at longer horizons, however. Results using the NAREIT index suggest that full sample OLS has the lowest mean absolute forecast error and is the preferred estimation method to obtain distant discount rates using CAPM. In contrast, for the ALL portfolio, while the full sample OLS is the most efficient method to estimate market risk premium factor loading at 1-year forecast horizon, $2^1/_2$-year rolling OLS gives the best estimate at 5-year forecast horizon.

We also consider the forecast dispersion for our portfolios. Panel B of Table 10 reports the standard deviation of absolute forecast error under the single-factor model. For near term forecast, the dispersions are similar for each portfolio. For more distant forecasts, however, how we estimate the factor loading has an effect. Results for the NAREIT index suggest that rolling regressions obtain lower dispersion. In contrast, results for the ALL portfolio show no consistent pattern. The full sample regression provides lower dispersion at 1-year horizon while rolling regressions provide lower dispersion at the 5-year forecast horizon.

To summarize, for discounting near term cash flows using CAPM CoEC, the estimation method - rolling or full sample OLS – has no effect on the forecast error and the accuracy of out-of-sample is comparable. For discounting distant cash flows, however, our analyses suggest that the choice of estimation strategy has a material effect. When we extend the sample period firm-by-firm as our robustness check, we find that the choice of estimate strategy is still important to obtain accurate distant CoEC. The actual recommended estimation method for a particular firm might however change with different sample period.[12]

1.5.2 Forecasts based on the three-factor model

In this section, we examine the predictive ability of factor loading under the three-factor model. The same method as in the preceding section is used to obtain the 1-month, 1-year and 5-year absolute forecast errors with factor loadings estimated using full sample OLS regression, $2^1/_2$-year rolling window, and 4-year rolling regression. The 1-month forecast error for the three-factor model is given by,

$$\varepsilon(1)_{t+1,i} = R_{t+1,i} - \hat{R}_{t+1,i} = R_{t+1,i} - b_{t,i}(\bar{R}_m - \bar{R}_f) - c_{t,i}(\overline{SMB}) - d_{t,i}(\overline{HML}) \tag{11}$$

[12] The complete set of results is available on request from the authors.

where loadings are estimated with rolling regressions. $b_{t,i}$ is the factor loading on the market factor at time t for firm i, $c_{t,i}$ is the loading on SMB, and $d_{t,i}$ is the factor loading on HML. $\overline{SMB}$ and $\overline{HML}$ represent, respectively, the long run average of the SMB and HML factors. Given $\varepsilon(1)_{t+1,i}$, we obtain 1-year and 5-year absolute forecast errors using (10).

The mean absolute forecast errors of the 60 firms are reported in Table 11. At 1-month forecast horizon, there appears to be no difference in accuracy between CoEC estimated via rolling or full sample OLS. The mean absolute forecast error averages 0.68 across sample firms. In essence, in estimating discount rates with the three-factor model for short term cash flows, a manager can be indifferent between $2^1/_2$-year rolling, 4-year rolling or full sample regressions.

Consistent with the results for the single-factor CAPM, we find that the choice of estimation method influences the accuracy of more distant discount rates; while there is variation across sample firms, no discernible pattern seems to emerge from the results. Using HCN as an example, the mean absolute forecast error at 1-year horizon is 1.562, 1.594 and 1.480 with loadings estimated using $2^1/_2$-year rolling, 4-year rolling or full sample OLS, respectively. The mean absolute forecast error at 5-year horizon is 4.075, 4.706 and 4.404 with loadings estimated using $2^1/_2$-year rolling, 4-year rolling or full sample OLS, respectively. As cash flows become more distant, recommended estimation method changes from full sample OLS (at 1-year forecast horizon) to rolling $2^1/_2$-year OLS (at 5-year horizon). Using HR as a second example, at 1-year forecast horizon, mean absolute forecast errors for $2^1/_2$-year rolling, 4-year rolling or full sample OLS are 1.956, 1.888 and 1.925, respectively; at 5-year horizon, mean absolute forecast errors are 3.628, 3.317 and 3.743 for $2^1/_2$-year rolling, 4-year rolling or full sample OLS estimation methods, respectively. So, as cash flows become more distant, we obtain a more accurate discount rate by using a 4-year rolling regression. Looking at the average of 60 firms on Table 11, full sample OLS appears to be the better method to estimate the three-factor model at 1-year forecast horizon with mean absolute forecast error of 2.430 (versus mean absolute forecast error of approximately 2.5 with rolling regressions). Rolling regressions however obtain lower mean absolute forecast error at 5-year forecast horizon.

Table 12 presents the standard deviations of absolute forecast errors for the 60 firms. For 1-month forecast horizon, one can be indifferent between the rolling and full sample OLS methods. For more distant forecasts, full sample OLS appears to have a lower dispersion of forecast error. For HCN: at 1-year forecast horizon, the standard deviations are 1.175, 1.173 and 1.067 using $2^{1}/_{2}$-year rolling, 4-year rolling or full sample OLS, respectively; at 5-year forecast horizon, the standard deviations are 1.913, 1.942 and 1.645 respectively. Averaging over 60 firms, the standard deviations at 1-year forecast horizon are 1.769, 1.747 and 1.738 respectively. At 5-year forecast horizons, the average standard deviations are 2.630, 2.598 and 2.455 using $2^{1}/_{2}$-year rolling, 4-year rolling or full sample OLS estimation method. There are exceptions to this overall pattern, however. Consider firm HR: at 1-year forecast horizon, the standard deviations are 1.347, 1.331 and 1.307 when we estimate regression slopes using $2^{1}/_{2}$-year rolling, 4-year rolling or full sample OLS respectively; at 5-year forecast horizon, the standard deviations are 2.138, 2.037 and 2.111 respectively.

We analyze the out-of-sample performance of portfolios using a similar method as discussed for the single-factor model. Under the three-factor model, however, we treat the SMB loading as time-invariant, while the market and HML loadings are allowed to vary. The 1-month forecast error model is given (11); the same methodology applies to full sample OLS regressions.

The results are presented in Table 13. Panel A reports the mean absolute forecast errors. In corroboration of previous results, the forecast errors at 1-month forecast horizon are similar regardless of estimation method. For distant horizons however, the estimation method influences forecast accuracy. For example, at 1-year forecast horizon, the CoEC of NAREIT index is better estimated using full sample OLS. On the other hand, at 5-year forecast horizon, the 4-year rolling regression shows less noise. Similarly, the ALL portfolio is better estimated by full sample OLS at 1-year forecast horizon, and by 4-year rolling regression at 5-year horizon. On the other hand, CoEC of Diversified portfolio is better estimated using full sample OLS at both 1-year and 5 year forecast horizons. We report standard deviations of absolute forecast errors in Panel B. As before, we find that all estimation methods have similar forecast dispersion at 1-month horizon. The story is different, however, for longer horizons. For example, the NAREIT index obtains lower forecast error dispersion under full sample regression. The ALL portfolio on the other hand obtains lower 1-year standard deviation when we use 4-year rolling regression and a lower 5-year standard deviation when we estimate via full sample OLS.

In sum, for valuation of near term cash flows by the three-factor model, the forecast error is comparable for the rolling and full sample estimation strategies. To value distant cash flows, however, forecast accuracy depends on the estimation method.[13]

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[13] Robustness checks using a firm-by-firm longer sample support our conclusions. Detailed results are available on request.

1.6 Conclusion and Issues for Future Research

The cost of equity literature on Equity REITs is rather limited. Our aim is to start a discussion of this important topic. We provide a comprehensive analysis of the cost of equity for a large sample of individual REITs, as well as equally-weighted portfolios of REITs formed on property types. We contend that our findings are new and constitute a significant step towards our understanding of risk attributes at the individual firm level.

The data reveal a number of surprises. For example, it is surprising to find such large variation in the cost of capital estimates across firms. This result is consistent whether one uses the single factor CAPM or the Fama-French three factor model. We were surprised also to see that these two premier pricing models yield such economically different point estimates. Another - not so surprising - result is that both CAPM beta and the Fama and French (1993) factor loadings may vary over time. The implication of these results is that valuation of future cash flows requires different strategies depending on the behavior of true loadings for each firm.

Our efforts leave unresolved a number of questions for future research. In present value analyses, should one use the single-factor CAPM or a three-factor model? The answer to this question may be different for different firms, depending on individual REIT characteristics. Another important issue is that currently available tests for the inter-temporal stability of factor loadings lack power or a statistical foundation for hypothesis testing. The informal method proposed by Fama and French (1997), while quite intuitive, cannot be used to answer the question “is this factor loading constant?” The CUSUM test appears to have minimal power. More sophisticated tests seem to be in order. Perhaps the Monte Carlo Markov Chain approach, advocated by Gelfand and Smith (1990), may offer a solution.

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Table 1 - Summary statistics of our 60-firm sample.

				Firm Returns				Volume	
Ticker	Firm Name	REIT Type	Mkt Val	Ave	Std	Skew	Kurt	Ave	Std
EPR	Entertainment Properties Trusts	Diversified	2,042	0.211	1.155	-0.110	8.648	1.559	3.609
WRE	Washington Real Estate Investment Trust	Diversified	1,807	0.112	0.781	-0.661	5.732	2.409	5.251
LXP	Lexington Realty Trust	Diversified	1,156	0.129	1.251	-0.029	15.270	3.687	8.439
CUZ	Cousins Properties Inc.	Diversified	665	0.041	1.106	-0.541	7.628	2.766	6.106
IRETS	Investors Real Estate Trusts	Diversified	610	0.077	0.545	0.006	3.594	0.922	2.136
HCP	HCP, Inc.	Health Care	16,928	0.188	0.965	-0.084	6.054	10.953	28.458
HCN	Health Care REIT, Inc.	Health Care	10,349	0.163	0.810	-0.095	3.411	5.588	13.883
OHI	Omega Healthcare Investors, Inc.	Health Care	1,996	0.148	1.666	0.180	5.576	3.058	6.756
HR	Healthcare Realty Trust	Health Care	1,447	0.113	0.936	-0.350	4.673	2.850	5.844
LTC	LTC Properties, Inc.	Health Care	936	0.168	1.067	-0.309	4.499	0.985	1.374
PLD	ProLogis Trust	Industrial	13,129	0.242	2.880	7.948	91.394	22.734	65.241
EGP	EastGroup Properties, Inc.	Industrial	1,177	0.154	0.806	-0.780	6.293	0.853	1.651
FR	First Industrial Realty Trust, Inc.	Industrial	886	0.102	1.512	-0.745	12.431	3.448	6.913
MNRTA	Monmouth Real Estate Investment Corp.	Industrial	359	0.139	0.624	0.704	6.698	0.262	0.549
HPT	Hospitality Properties Trust	Lodging	2,839	0.137	1.051	-0.687	9.247	4.999	11.261
LHO	LaSalle Hotel Properties	Lodging	2,028	0.224	1.687	1.898	21.447	2.852	6.839
FCH	FelCor Lodging Trust Incorporated	Lodging	380	0.075	2.117	0.724	7.139	4.262	8.044
EQR	Equity Residential Properties Trust	Multi-family	16,917	0.166	0.871	-0.566	4.690	13.660	28.908
AVB	AvalonBay Communities Inc.	Multi-family	12,418	0.184	0.869	-0.513	4.504	4.940	11.692
ESS	Essex Property Trust, Inc.	Multi-family	4,795	0.195	0.830	-0.195	3.581	1.880	4.251
CPT	Camden Property Trust	Multi-family	4,441	0.164	0.956	-0.525	5.365	3.456	7.333
BRE	BRE Properties, Inc.	Multi-family	3,801	0.144	0.897	-0.522	5.469	3.281	7.025
HME	Home Properties Inc.	Multi-family	2,779	0.153	0.809	-0.876	6.298	1.073	1.425
AIV	Apartment Investment & Management Co.	Multi-family	2,770	0.120	1.242	-0.966	9.268	7.875	17.651
MAA	Mid-America Apartment Communities, Inc.	Multi-family	2,366	0.173	0.786	-0.663	5.649	1.418	3.088
PPS	Post Properties, Inc.	Multi-family	2,268	0.111	1.000	-0.449	4.840	3.225	6.054
CLP	Colonial Properties Trust	Multi-family	1,823	0.160	1.436	2.270	28.998	2.695	5.764
AEC	Associated Estates Realty Corporation	Multi-family	675	0.173	1.171	-0.115	5.968	0.910	1.699

Table 1 - Continued

				Firm Returns				Volume	
Ticker	Firm Name	REIT Type	Mkt Val	Ave	Std	Skew	Kurt	Ave	Std
BXP	Boston Properties, Inc.	Office	14,726	0.179	0.906	0.183	8.334	6.819	15.618
SLG	SL Green Realty Corp.	Office	5,741	0.219	1.451	-0.004	10.921	4.958	13.780
ARE	Alexandria Real Estate Equities Inc.	Office	4,273	0.150	1.060	-0.178	12.882	2.132	5.512
DRE	Duke-Weeks Realty Corporation	Office	3,048	0.086	1.294	1.675	20.914	10.020	23.984
CLI	Mack-Cali Realty Corporation	Office	2,326	0.097	0.976	0.394	6.969	4.098	8.489
KRC	Kilroy Realty Corporation	Office	2,226	0.139	1.009	-0.553	5.204	2.436	4.959
HIW	Highwoods Properties, Inc.	Office	2,153	0.116	0.905	-0.629	4.208	3.868	7.145
OFC	Corporate Office Properties Trust	Office	1,530	0.170	0.859	-0.486	4.051	2.589	6.138
BDN	Brandywine Realty Trust	Office	1,288	0.131	1.737	3.453	33.582	5.623	13.508
PKY	Parkway Properties, Inc.	Office	217	0.037	1.204	-0.752	9.316	0.618	1.138
SPG	Simon Property Group, Inc.	Retail	37,888	0.212	1.010	0.165	10.412	11.967	30.576
MAC	Macerich Company, The	Retail	6,676	0.270	2.185	5.651	60.772	5.700	15.220
KIM	Kimco Realty Corporation	Retail	6,609	0.132	1.231	0.586	14.390	15.380	43.703
FRT	Federal Realty Investment Trust	Retail	5,762	0.185	0.801	-1.011	5.946	3.113	6.776
O	Realty Income Corporation	Retail	4,657	0.168	0.689	-0.016	3.620	4.004	10.135
TCO	Taubman Centers, Inc.	Retail	3,597	0.216	1.045	-0.348	7.230	3.750	7.321
REG	Regency Realty Corporation	Retail	3,383	0.142	1.001	-0.208	10.359	3.820	9.496
DDR	Developers Diversified Realty Corporation	Retail	3,371	0.158	1.649	0.785	20.190	10.638	27.935
WRI	Weingarten Realty Investors	Retail	2,637	0.128	1.222	1.209	16.292	4.552	10.771
SKT	Tanger Factory Outlet Centers, Inc.	Retail	2,542	0.222	0.791	0.643	8.192	1.972	4.537
CBL	CBL & Associates Properties, Inc.	Retail	2,329	0.290	2.757	6.818	71.222	5.781	14.883
EQY	Equity One, Inc.	Retail	1,947	0.148	0.799	-0.467	4.128	1.942	4.297
ALX	Alexander's Inc.	Retail	1,889	0.180	1.127	-0.356	5.302	0.164	0.195
GRT	Glimcher Realty Trust	Retail	989	0.177	1.609	0.262	11.188	2.551	5.517
AKR	Acadia Realty Trust	Retail	858	0.192	0.858	0.002	8.345	1.131	2.400
BFS	Saul Centers, Inc.	Retail	683	0.158	0.939	0.448	7.087	0.389	0.571
PEI	Pennsylvania Real Estate Investment Trust	Retail	581	0.172	1.874	2.537	24.900	2.222	5.071
RPT	Ramco-Gershenson Properties Trust	Retail	383	0.136	1.359	0.074	18.018	0.743	1.666

Table 1 – Continued

Ticker	Firm Name	REIT Type	Mkt Val	Firm Returns				Volume	
				Ave	Std	Skew	Kurt	Ave	Std
OLP	One Liberty Properties, Inc.	Retail	240	0.184	1.342	1.433	19.076	0.212	0.491
ADC	Agree Realty Corp	Retail	240	0.156	1.082	0.118	7.898	0.250	0.432
RPI	Roberts Realty Investors, Inc.	Retail	13	0.051	1.322	1.282	14.332	0.027	0.054
SSS	Sovran Self Storage, Inc.	Self Storage	1,189	0.145	0.900	-0.259	6.056	0.766	1.356

Notes: Table 1 reports summary statistics of the 60 firms in our sample. The period is January 1999 to December 2011. *REIT Type* denotes property focus. *Mkt Val* stands for market value. The market value is in millions of dollars as of 2011 Q4. We obtain market value as price per share multiplied by the number of shares outstanding obtained from CRSP. Firm returns signify annualized 1-month holding-period-return obtained from CRSP. We annualize returns by multiplying with 12. *Ave, Std, Skew* and *Kurt* signify average, standard deviation, skewness and kurtosis respectively. *Volume* is the monthly trading volume in millions of shares.

Table 2 – Summary statistics of REIT Type portfolios, NAREIT index and factors.

Panel A: Summary statistics of REIT Type portfolios and NAREIT index, 01/1999 to 12/2011

		Returns			
Number of firms	Property type	Ave	Std	Skew	Kurt
5	Diversified	0.114	0.968	-0.267	8.175
5	Health Care	0.156	1.089	-0.132	4.843
4	Industrial	0.159	1.455	1.782	29.204
3	Lodging	0.145	1.618	0.645	12.611
11	Multi-family	0.159	0.988	-0.284	7.694
10	Office	0.132	1.140	0.310	11.638
21	Retail	0.175	1.271	0.934	16.614
1	Self Storage	0.145	0.900	-0.259	6.056
60	ALL	0.155	1.180	0.440	12.928
NAREIT Equity REITs index		0.128	0.809	-0.793	9.278

Panel B: Summary statistics of market risk premium, SMB and HML factors from 1927 to 2011

	Returns		
Factors	Ave	Std	Std. Err.
R_f	0.036	0.031	0.003
R_m - R_f	0.079	0.208	0.023
SMB	0.037	0.142	0.015
HML	0.047	0.139	0.015

Notes: Table 2 reports summary statistics of REIT Type portfolios, NAREIT Equity REITs index and factors. Panel A reports summary statistics of annualized REIT Type portfolios and NAREIT index. To obtain REIT Type portfolios, we group our 60-firm sample based on property type and form equal-weighted portfolios. In addition, we also report an equal-weighted portfolio based on all 60 firms in our sample. *ALL* signifies portfolio obtained using all firms. NAREIT denotes National Association of Real Estate Investment Trusts. NAREIT Equity REITs index data was obtained from NAREIT's website. We annualize by multiplying monthly returns with 12. Sample period is January 1999 to December 2011. Panel B presents summary statistics of annual factors from 1927 to 2011. We obtained factors from Dr. Kenneth French's website. *Ave*, *Std*, *Skew* and *Kurt* denote average, standard deviation, skewness and kurtosis respectively. S*td. Err.* signifies standard deviation divided by square root of the number of data.

Table 3 – CAPM cost of equity capital for our 60-firm sample.

Ticker	a	b	R^2	CoEC
EPR	0.015***	0.536***	0.165	7.882
WRE	0.008**	0.397***	0.138	6.782
LXP	0.008*	0.479***	0.140	7.429
CUZ	0.002	0.566***	0.172	8.119
IRETS	0.004	0.222***	0.072	5.389
HCP	0.014***	0.432***	0.116	7.060
HCN	0.011**	0.396***	0.109	6.770
OHI	0.006	0.762***	0.117	9.677
HR	0.007	0.575***	0.181	8.193
LTC	0.011**	0.495***	0.105	7.557
PLD	0.008*	0.730***	0.305	9.425
EGP	0.012***	0.466***	0.179	7.330
FR	0.006	0.696***	0.194	9.153
MNRTA	0.008**	0.253***	0.091	5.636
HPT	0.009*	0.599***	0.196	8.380
LHO	0.016***	1.013***	0.329	11.668
FCH	-0.009	1.594***	0.387	16.275
EQR	0.012***	0.617***	0.240	8.528
AVB	0.014***	0.638***	0.270	8.688
ESS	0.013***	0.535***	0.198	7.877
CPT	0.012***	0.631***	0.265	8.633
BRE	0.011**	0.506***	0.181	7.647
HME	0.012***	0.402***	0.130	6.816
AIV	0.007	0.725***	0.239	9.378
MAA	0.012***	0.538***	0.235	7.900
PPS	0.007	0.672***	0.260	8.958
CLP	0.010**	0.427***	0.157	7.016
AEC	0.012**	0.493***	0.107	7.545
BXP	0.013***	0.589***	0.258	8.305
SLG	0.014***	0.801***	0.278	9.985
ARE	0.010**	0.414***	0.140	6.911
DRE	0.002	0.681***	0.260	9.032
CLI	0.004	0.546***	0.200	7.961
KRC	0.010**	0.713***	0.247	9.284
HIW	0.007	0.616***	0.196	8.520
OFC	0.012**	0.533***	0.168	7.857
BDN	0.001	0.725***	0.228	9.378
PKY	-0.000	0.735***	0.222	9.458
SPG	0.015***	0.496***	0.158	7.568
MAC	0.012**	0.606***	0.172	8.437
KIM	0.007	0.582***	0.198	8.247
FRT	0.015***	0.408***	0.139	6.869
O	0.011***	0.341***	0.113	6.336
TCO	0.016***	0.649***	0.264	8.775
REG	0.010**	0.466***	0.162	7.325
DDR	0.010*	0.709***	0.212	9.254
WRI	0.006	0.451***	0.146	7.211
SKT	0.015***	0.405***	0.138	6.843

Table 3 – Continued

Ticker	a	b	R^2	CoEC
CBL	0.005	0.725***	0.197	9.381
EQY	0.010**	0.570***	0.244	8.156
ALX	0.014**	0.747***	0.227	9.558
GRT	0.012*	0.869***	0.234	10.524
AKR	0.014***	0.447***	0.177	7.174
BFS	0.011**	0.328***	0.065	6.230
PEI	0.007	0.756***	0.210	9.627
RPT	0.008	0.529***	0.141	7.826
OLP	0.010***	0.480***	0.217	7.440
ADC	0.010**	0.422***	0.110	6.980
RPI	-0.001	0.303***	0.046	6.031
SSS	0.011**	0.387***	0.107	6.698
Ave	0.009	0.574	0.186	8.182

Notes: Table 3 reports cost of equity capital (CoEC) based on the CAPM

$$R_t - R_{f,t-1} = \mathrm{a} + \mathrm{b}\left(R_{m,t} - R_{f,t-1}\right)$$

of our 60-firm sample. The period is January 1999 to December 2011. CoECs are in percent per year. The factor loadings are estimated via ordinary least squares for each firm. To calculate the unconditional CoEC, we use the average annual risk free rate and market risk premium from 1927 to 2011. The average annual risk free rate is 3.63% and the average annual market risk premium is 7.94%. ***,** and * signify statistically different from zero at 1%, 5% and 10% critical value. In addition, we report the regression coefficients. Coefficient *a* is the regression intercept and measures the goodness of fit associated with the model. Coefficient *b* is the regression loading associated with market risk premium factor $R_m - R_f$ and measures the amount of risk associated with the factor. *Ave* is average of 60 firms.

Table 4 – Fama and French (1993) three-factor cost of equity capital for our 60-firm sample.

Ticker	a	b	c	d	R^2	CoEC
EPR	0.011**	0.474***	0.593***	0.626***	0.322	12.523
WRE	0.003	0.363***	0.519***	0.719***	0.397	11.814
LXP	0.004	0.475***	0.447***	0.829***	0.357	12.958
CUZ	-0.002	0.547***	0.427***	0.667***	0.303	12.688
IRETS	0.002	0.210***	0.226**	0.334***	0.164	7.703
HCP	0.012**	0.480***	0.085	0.569***	0.219	10.445
HCN	0.008*	0.389***	0.305**	0.531***	0.212	10.347
OHI	0.001	0.768***	0.407*	0.837***	0.188	15.177
HR	0.003	0.595***	0.317**	0.778***	0.344	13.190
LTC	0.008	0.503***	0.265	0.580***	0.177	11.336
PLD	0.006	0.781***	0.033	0.487***	0.377	12.252
EGP	0.008**	0.467***	0.301***	0.587***	0.324	11.216
FR	0.001	0.725***	0.363**	0.944***	0.371	15.178
MNRTA	0.006**	0.249***	0.187**	0.330***	0.171	7.850
HPT	0.006	0.620***	0.196	0.556***	0.279	11.898
LHO	0.011**	1.011***	0.441***	0.833***	0.443	17.210
FCH	-0.018***	1.637***	0.673***	1.667***	0.596	26.974
EQR	0.009**	0.664***	0.180	0.738***	0.411	13.048
AVB	0.010***	0.663***	0.255**	0.705***	0.432	13.158
ESS	0.010**	0.523***	0.374***	0.622***	0.340	12.092
CPT	0.008**	0.630***	0.340***	0.656***	0.412	12.981
BRE	0.007*	0.508***	0.354***	0.701***	0.358	12.278
HME	0.008**	0.380***	0.377***	0.545***	0.264	10.603
AIV	0.002	0.719***	0.488***	0.892***	0.427	15.339
MAA	0.010**	0.527***	0.243**	0.377***	0.298	10.487
PPS	0.004	0.688***	0.266**	0.652***	0.382	13.150
CLP	0.006*	0.390***	0.445***	0.553***	0.319	10.970
AEC	0.007	0.442***	0.592***	0.711***	0.247	12.665
BXP	0.010***	0.616***	0.158	0.536***	0.363	11.637
SLG	0.009*	0.769***	0.539***	0.776***	0.425	15.378
ARE	0.007*	0.389***	0.405***	0.580***	0.296	10.946
DRE	-0.002	0.694***	0.333***	0.753***	0.418	13.916
CLI	0.001	0.592***	0.144	0.664***	0.347	11.994
KRC	0.005	0.653***	0.618***	0.688***	0.401	14.325
HIW	0.003	0.638***	0.333**	0.824***	0.369	13.810
OFC	0.007*	0.491***	0.535***	0.680***	0.332	12.701
BDN	-0.004	0.710***	0.539***	0.918***	0.421	15.576
PKY	-0.006	0.672***	0.677***	0.781***	0.385	15.133
SPG	0.011***	0.508***	0.314***	0.708***	0.318	12.164
MAC	0.007	0.575***	0.548***	0.799***	0.340	13.981
KIM	0.004	0.590***	0.276**	0.604***	0.305	12.181
FRT	0.012***	0.413***	0.308***	0.632***	0.306	11.023
O	0.007**	0.319***	0.386***	0.560***	0.284	10.222
TCO	0.013***	0.668***	0.232**	0.612***	0.380	12.674
REG	0.006	0.448***	0.354***	0.532***	0.279	10.996
DDR	0.007	0.761***	0.165	0.760***	0.333	13.869
WRI	0.003	0.472***	0.241**	0.639***	0.291	11.284
SKT	0.013***	0.428***	0.135	0.453***	0.223	9.660

Table 4 – Continued

Ticker	a	b	c	d	R^2	CoEC
CBL	-0.001	0.641***	0.847***	0.934***	0.416	16.236
EQY	0.007*	0.556***	0.328***	0.517***	0.352	11.695
ALX	0.010*	0.698***	0.476***	0.508***	0.299	13.318
GRT	0.007	0.848***	0.561***	0.906***	0.370	16.699
AKR	0.012***	0.453***	0.205*	0.446***	0.266	10.081
BFS	0.007	0.303***	0.410***	0.585***	0.183	10.301
PEI	0.001	0.729***	0.601***	0.936***	0.386	16.046
RPT	0.005	0.514***	0.362**	0.578***	0.232	11.768
OLP	0.007**	0.468***	0.308***	0.488***	0.338	10.775
ADC	0.006	0.381***	0.489***	0.596***	0.245	11.260
RPI	-0.003	0.295**	0.237	0.393**	0.088	8.697
SSS	0.007*	0.368***	0.358***	0.533***	0.219	10.381
Ave	0.005	0.568	0.369	0.666	0.322	12.638

Notes: Table 4 reports cost of equity capital (CoEC) based on Fama and French (1993) three-factor model

$$R_t - R_{f,t-1} = \mathrm{a} + \mathrm{b}\left(R_{m,t} - R_{f,t-1}\right) + \mathrm{c}\left(SMB_t\right) + \mathrm{d}\left(HML_t\right)$$

of our 60-firm sample. The period is January 1999 to December 2011. Factor loadings are estimated via ordinary least squares for each firm. CoECs are in percent per year. To calculate the unconditional CoEC, we use the average annual risk free rate, market risk premium, SMB and HML from 1927 to 2011. The average annual risk free rate is 3.63%, the average annual market risk premium is 7.94%, the average annual SMB is 3.66% and the average annual HML is 4.74%. ***,** and * signify statistically different from zero at 1%, 5% and 10% critical value. We also report the regression coefficients. Coefficient *a* is the intercept of the regression and measures the goodness of fit associated with the model. Loadings *b*, *c* and *d* are amounts of risk associated with market risk premium factor $R_m - R_f$, size factor SMB and financial distress factor HML respectively. *Ave* is average of 60 firms.

Table 5 – Unconditional cost of equity capital for REIT Type portfolios and NAREIT index.

Panel A: CAPM

Property type	a	b	R^2	CoEC
Diversified	0.007**	0.440***	0.233	7.120
Health Care	0.010**	0.532***	0.187	7.852
Industrial	0.008***	0.536***	0.310	7.886
Lodging	0.005	1.069***	0.419	12.108
Multi-family	0.011***	0.562***	0.291	8.090
Office	0.007**	0.635***	0.314	8.669
Retail	0.010***	0.538***	0.310	7.895
Self Storage	0.011**	0.387***	0.107	6.698
ALL	0.009***	0.574***	0.351	8.182
NAREIT Index	0.007*	0.862***	0.396	10.466

Panel B: Fama and French (1993) three-factor

Property type	a	b	c	d	R^2	CoEC
Diversified	0.003	0.414***	0.442***	0.635***	0.508	11.537
Health Care	0.006	0.547***	0.276**	0.659***	0.329	12.099
Industrial	0.005*	0.555***	0.221***	0.587***	0.493	11.624
Lodging	-0.000	1.089***	0.437***	1.019***	0.609	18.694
Multi-family	0.007**	0.558***	0.356***	0.650***	0.492	12.434
Office	0.003	0.622***	0.428***	0.720***	0.527	13.542
Retail	0.007**	0.527***	0.371***	0.628***	0.534	12.140
Self Storage	0.007*	0.368***	0.358***	0.533***	0.219	10.381
ALL	0.005**	0.568***	0.369***	0.666***	0.596	12.638
NAREIT Index	0.002	0.874***	0.442***	0.962***	0.643	16.739

Notes: Table 5 reports cost of equity capital (CoEC) based on CAPM

$$R_t - R_{f,t-1} = \mathrm{a} + \mathrm{b}\left(R_{m,t} - R_{f,t-1}\right)$$

and Fama and French (1993) three-factor model

$$R_t - R_{f,t-1} = \mathrm{a} + \mathrm{b}\left(R_{m,t} - R_{f,t-1}\right) + \mathrm{c}\left(SMB_t\right) + \mathrm{d}\left(HML_t\right)$$

of REIT Type portfolios and NAREIT index. *REIT Type* portfolios are equal-weighted portfolios formulated by grouping our sample by property types. *ALL portfolio* is an equal-weighted portfolio of all 60 firms in our sample. Panel A reports results of CAPM. Coefficient *a* is the intercept of the regression and signifies goodness of fit associated with the model. Coefficient *b* is the loading of market risk premium factor and measures the amount of risk associated with the factor. Fama and French (1993) results are presented in Panel B. In Panel B, we also present the regression coefficients *c* and *d* associated with size factor SMB and financial distress factor HML; the coefficients measure the amount of risk associated with the factors. The period is January 1999 to December 2011. CoECs are in percent per year. Models are estimated via ordinary least squares. To calculate the CoEC, we use the average annual risk free rate, market risk premium, SMB and HML from 1927 to 2011. The average annual risk free rate is 3.63%, the average annual market risk premium is 7.94%, the average annual SMB is 3.66% and the average annual HML is 4.74%. ***,** and * signify statistically different from zero at 1%, 5% and 10% critical value.

Table 6 - Implied variation of the true factor loadings in 60-firm sample.

	Panel A: CAPM	Panel B: Fama and French (1993) three-factor model		
Ticker	$\mathrm{Var}(\beta)$	$\mathrm{Var}(\beta)$	$\mathrm{Var}(\chi)$	$\mathrm{Var}(\delta)$
EPR	0.098	0.025	-0.068	-0.041
WRE	0.185	0.046	-0.077	0.090
LXP	0.091	-0.004	-0.064	0.002
CUZ	0.077	-0.039	-0.121	-0.140
IRETS	0.078	0.028	-0.013	-0.020
HCP	0.110	0.062	-0.069	0.122
HCN	0.134	0.062	-0.073	-0.060
OHI	0.037	-0.119	-0.072	0.068
HR	0.102	0.056	0.007	0.003
LTC	0.078	-0.028	0.016	-0.024
PLD	0.150	0.091	-0.123	-0.001
EGP	0.091	-0.010	-0.082	-0.039
FR	0.105	-0.052	-0.178	0.019
MNRTA	-0.006	-0.021	0.014	0.028
HPT	0.026	-0.043	-0.068	-0.014
LHO	0.164	0.132	-0.164	-0.090
FCH	0.259	0.015	-0.082	0.204
EQR	0.074	0.012	-0.054	0.374
AVB	0.100	0.003	-0.084	0.002
ESS	0.047	-0.036	-0.047	0.263
CPT	0.070	-0.027	-0.120	-0.056
BRE	0.066	-0.024	-0.113	0.018
HME	0.094	0.036	-0.112	0.146
AIV	0.102	-0.011	-0.091	-0.104
MAA	0.028	-0.017	-0.084	-0.002
PPS	0.047	-0.037	-0.109	0.106
CLP	0.097	-0.009	0.006	-0.041
AEC	-0.036	-0.083	-0.125	-0.146
BXP	0.060	-0.009	-0.068	-0.011
SLG	0.183	0.063	-0.168	-0.099
ARE	0.144	0.040	-0.088	-0.042
DRE	0.061	-0.031	-0.061	-0.039
CLI	0.088	0.033	-0.013	-0.053
KRC	0.086	-0.029	-0.036	0.061
HIW	0.155	0.027	-0.123	-0.130
OFC	0.121	-0.030	-0.061	-0.057
BDN	0.141	-0.027	-0.096	-0.164
PKY	0.017	-0.090	-0.097	-0.096
SPG	0.165	0.119	-0.035	-0.014
MAC	0.114	0.047	-0.029	-0.050
KIM	0.083	-0.019	-0.158	-0.021
FRT	0.127	0.022	-0.066	0.022
O	0.127	0.074	-0.058	-0.017
TCO	0.033	-0.049	-0.137	-0.028
REG	0.156	0.077	-0.103	-0.020

Table 6 - Continued

	Panel A: CAPM	Panel B: Fama and French (1993) three-factor model		
Ticker	$\mathrm{Var}(\beta)$	$\mathrm{Var}(\beta)$	$\mathrm{Var}(\chi)$	$\mathrm{Var}(\delta)$
DDR	0.145	0.054	-0.102	-0.077
WRI	0.159	0.091	-0.099	0.072
SKT	0.101	-0.011	0.011	-0.034
CBL	0.160	0.061	-0.145	-0.167
EQY	0.119	0.052	-0.101	-0.099
ALX	0.133	0.025	-0.048	-0.199
GRT	0.267	0.062	-0.175	-0.024
AKR	0.017	-0.024	-0.057	-0.017
BFS	0.099	-0.070	-0.135	-0.002
PEI	0.250	0.059	-0.058	-0.134
RPT	0.084	0.047	-0.002	-0.065
OLP	0.088	0.077	0.028	-0.026
ADC	0.091	0.009	-0.115	-0.089
RPI	0.092	0.046	-0.044	-0.158
SSS	0.093	0.028	-0.081	-0.030
Ave	0.104	0.012	-0.078	-0.019

Notes: Table 6 reports implied variations in the true factor loadings of our 60-firm sample. The period is January 1999 to December 2011. For each firm, we obtain time-series estimates of factor loadings for CAPM as shown in equation (1) and then calculate the difference between time-series variance of the estimated loading and sample average of the square of the standard error:

$$Var(\beta) = Var(b) - \overline{s^2\left((X'X)^{-1}\right)_{2,2}}$$

as shown in equation (3). For Fama French (1993) three-factor model, we obtain time-series estimates of factor loadings as shown in equation (2) and then calculate

$$\mathrm{Var}(\beta) = \mathrm{Var}(b) - \overline{\mathrm{s}^2\left(\left(X'X\right)^{-1}\right)_{2,2}}$$
$$\mathrm{Var}(\chi) = \mathrm{Var}(c) - \overline{\mathrm{s}^2\left(\left(X'X\right)^{-1}\right)_{3,3}}$$
$$\mathrm{Var}(\delta) = \mathrm{Var}(d) - \overline{\mathrm{s}^2\left(\left(X'X\right)^{-1}\right)_{4,4}}$$

from equation (4). We estimate time-series of factor loadings using ordinary least squares (OLS) with 3 year rolling window. *Ave* is average of 60 firms.

Table 7 - Implied variation of the true factor loadings in three-year rolling regression

	Panel A: CAPM	Panel B: Fama and French (1993) three-factor model		
Property type	$\mathrm{Var}(\beta)$	$\mathrm{Var}(\beta)$	$\mathrm{Var}(\chi)$	$\mathrm{Var}(\delta)$
Diversified	0.109	0.031	-0.038	-0.004
Health Care	0.116	0.032	-0.012	-0.021
Industrial	0.069	0.006	-0.053	0.006
Lodging	0.117	0.021	-0.064	0.084
Multi-family	0.070	-0.004	-0.062	0.037
Office	0.110	0.006	-0.066	-0.056
Retail	0.121	0.041	-0.060	-0.016
Self Storage	0.093	0.028	-0.081	-0.030
ALL	0.105	0.023	-0.048	-0.002
NAREIT	0.257	0.073	-0.035	0.041

Notes: Table 7 reports implied variations in the true factor loadings of our portfolios and NAREIT index. Firms were grouped by REIT Types and equal-weighted portfolios were created. *ALL portfolio* is an equal-weighted portfolio consisting of all 60 firms. *REIT Type* portfolios are equal-weighted portfolios formulated by grouping our sample by property types. The period is January 1999 to December 2011. For each firm, we obtain time-series estimates of factor loadings for CAPM as shown in equation (1) and then calculate the difference between time-series variance of the estimated loading and sample average of the square of the standard error:

$$Var(\beta) = Var(b) - \overline{s^2\left((X'X)^{-1}\right)_{2,2}}$$

as in equation (3). For Fama French (1993) three-factor model, we obtain time-series estimates of factor loadings as shown in equation (2) and then calculate

$$\mathrm{Var}(\beta) = \mathrm{Var}(b) - \overline{\mathrm{s}^2\left(\left(X'X\right)^{-1}\right)_{2,2}}$$
$$\mathrm{Var}(\chi) = \mathrm{Var}(c) - \overline{\mathrm{s}^2\left(\left(X'X\right)^{-1}\right)_{3,3}}$$
$$\mathrm{Var}(\delta) = \mathrm{Var}(d) - \overline{\mathrm{s}^2\left(\left(X'X\right)^{-1}\right)_{4,4}}$$

from equation (4). We estimate time-series of factor loadings using ordinary least squares (OLS) with 3 year rolling window.

Table 8 – CAPM mean absolute forecast errors for our 60-firm sample.

	Estimation method: 2.5 yr rolling OLS			Estimation method: 4 yr rolling OLS			Estimation method: Full sample OLS		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Ticker	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
EPR	0.666	2.527	5.573	0.667	2.584	6.465	0.665	2.537	7.351
WRE	0.548	1.723	1.439	0.551	1.753	1.885	0.549	1.655	2.657
LXP	0.644	2.225	2.266	0.646	2.201	2.221	0.640	2.161	2.295
CUZ	0.728	2.458	6.955	0.729	2.481	6.607	0.723	2.266	5.833
IRETS	0.437	0.739	1.223	0.437	0.743	0.971	0.439	0.749	1.140
HCP	0.651	1.813	4.977	0.650	1.867	5.780	0.651	1.946	7.003
HCN	0.571	1.584	4.212	0.572	1.660	5.044	0.571	1.737	6.550
OHI	0.814	2.810	6.376	0.818	2.886	7.186	0.816	2.820	8.056
HR	0.670	1.999	3.108	0.670	1.986	2.740	0.667	1.868	1.785
LTC	0.690	2.377	5.144	0.694	2.458	5.994	0.692	2.395	6.902
PLD	0.715	2.802	4.582	0.718	2.890	4.131	0.708	2.652	3.389
EGP	0.558	1.892	2.951	0.560	2.002	3.641	0.556	1.909	4.357
FR	0.820	3.176	5.339	0.822	3.248	4.887	0.816	3.122	4.960
MNRTA	0.402	1.088	1.415	0.401	1.087	1.331	0.402	1.121	1.366
HPT	0.677	2.324	3.213	0.679	2.399	2.984	0.677	2.327	2.552
LHO	0.877	3.640	2.815	0.875	3.571	2.862	0.870	3.488	2.334
FCH	1.301	4.681	17.071	1.301	4.805	16.683	1.298	4.551	16.138
EQR	0.626	2.666	2.807	0.627	2.713	3.291	0.628	2.660	4.086
AVB	0.649	3.153	4.762	0.651	3.199	5.568	0.648	3.085	6.326
ESS	0.621	2.727	3.505	0.621	2.778	4.195	0.619	2.700	4.708
CPT	0.668	2.708	1.858	0.670	2.769	2.149	0.668	2.683	2.537
BRE	0.609	2.382	3.087	0.610	2.456	3.888	0.610	2.348	4.396
HME	0.607	1.829	3.869	0.607	1.873	4.756	0.607	1.829	5.609
AIV	0.783	2.768	1.865	0.783	2.786	2.323	0.783	2.768	2.508
MAA	0.608	2.406	3.505	0.611	2.476	3.976	0.609	2.390	4.741
PPS	0.686	3.233	2.243	0.689	3.319	2.247	0.689	3.262	2.365
CLP	0.571	2.789	2.023	0.571	2.820	2.115	0.567	2.704	2.483
AEC	0.730	3.108	7.570	0.730	3.091	7.923	0.728	3.060	7.628
BXP	0.621	3.001	5.232	0.622	3.026	5.889	0.616	2.912	6.167
SLG	0.805	4.290	3.677	0.808	4.380	4.251	0.800	4.228	4.436
ARE	0.591	1.981	2.545	0.592	2.055	3.390	0.588	1.921	4.299
DRE	0.700	2.432	7.266	0.700	2.464	6.599	0.695	2.318	6.209
CLI	0.672	1.920	4.809	0.671	1.925	4.367	0.667	1.759	3.601
KRC	0.710	3.316	2.996	0.710	3.349	3.195	0.703	3.186	2.817
HIW	0.685	1.884	2.228	0.687	1.989	2.966	0.684	1.809	3.233
OFC	0.666	2.838	4.214	0.670	2.924	4.719	0.663	2.706	4.654
BDN	0.796	2.755	9.567	0.796	2.782	8.630	0.791	2.559	8.460
PKY	0.791	2.331	7.133	0.790	2.351	6.815	0.788	2.239	6.881
SPG	0.654	2.853	4.705	0.657	2.978	5.712	0.656	2.870	6.684
MAC	0.738	3.265	3.173	0.741	3.347	3.058	0.737	3.203	2.857

Table 8 – Continued

Ticker	Estimation method: 2.5 yr rolling OLS			Estimation method: 4 yr rolling OLS			Estimation method: Full sample OLS		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
KIM	0.718	3.058	5.230	0.721	3.149	5.031	0.711	2.965	4.359
FRT	0.607	2.480	6.638	0.609	2.574	7.554	0.604	2.463	8.324
O	0.510	1.898	2.096	0.513	1.943	2.698	0.512	1.888	3.837
TCO	0.670	3.156	8.053	0.673	3.246	8.611	0.671	3.181	9.003
REG	0.654	2.422	3.568	0.656	2.519	3.568	0.647	2.289	3.172
DDR	0.824	3.614	3.489	0.827	3.684	3.152	0.817	3.480	2.702
WRI	0.665	2.237	4.245	0.666	2.304	3.877	0.663	2.121	2.988
SKT	0.539	2.153	6.826	0.541	2.228	7.729	0.540	2.248	9.014
CBL	0.890	4.005	10.661	0.892	4.038	10.119	0.885	3.825	9.105
EQY	0.625	2.096	2.861	0.625	2.143	2.555	0.621	1.971	1.940
ALX	0.813	3.358	7.528	0.811	3.348	8.779	0.808	3.296	9.755
GRT	0.937	4.194	4.733	0.933	4.116	4.121	0.931	3.955	4.874
AKR	0.577	2.308	5.203	0.579	2.342	5.628	0.575	2.282	6.118
BFS	0.671	2.187	3.444	0.673	2.373	4.186	0.670	2.308	5.280
PEI	0.886	3.434	7.063	0.887	3.435	6.179	0.878	3.278	5.784
RPT	0.731	2.355	2.836	0.734	2.406	2.342	0.726	2.249	2.063
OLP	0.541	1.902	1.547	0.542	1.982	1.873	0.538	1.875	2.232
ADC	0.633	2.411	2.288	0.635	2.525	2.231	0.632	2.400	2.126
RPI	0.681	2.839	7.692	0.678	2.825	7.759	0.680	2.929	8.151
SSS	0.602	2.547	1.808	0.603	2.593	2.305	0.598	2.449	2.740
Ave	0.685	2.619	4.518	0.687	2.671	4.729	0.683	2.566	4.932

Notes: Table 8 presents mean absolute forecast errors using CAPM as cost of equity capital (CoEC) model for our 60-firm sample. We examine 3 estimation methods: 2.5 year rolling window ordinary least squares (OLS), 4 year rolling window OLS and full sample OLS. The estimation period is January 1999 to December 2011. The forecast period analyzed were 1 month forecast, 1 year forecast and 5 year forecast. For each firm, we obtained CAPM factor loadings estimated using 2.5 year rolling, 4 year rolling and full sample OLS. Given the factor loadings, we calculated estimated CAPM CoECs with long run average of market risk premium as factor. Using 1927 to 2011 as our sample, annual long run average of market risk premium is 7.94% and annual long run average of risk free rate is 3.63%. We then obtained monthly forecast errors as the difference between 1-month ahead realized annualized return and our estimated CAPM CoECs. Given the monthly forecast errors, we obtained 1 month, 1 year and 5 years absolute forecast errors. We report the mean of absolute forecast errors for each firm. *Ave* is average of 60 firms.

Table 9 – CAPM standard deviation of absolute forecast errors for our 60-firm sample.

	Estimation method: 2.5 yr rolling OLS			Estimation method: 4 yr rolling OLS			Estimation method: Full sample OLS		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Ticker	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
EPR	0.479	1.750	2.399	0.483	1.790	2.441	0.483	1.813	2.355
WRE	0.376	1.058	1.043	0.373	1.074	1.373	0.371	1.030	1.326
LXP	0.500	1.552	1.621	0.498	1.589	1.489	0.504	1.656	1.800
CUZ	0.506	1.553	3.555	0.506	1.533	3.261	0.505	1.426	2.870
IRETS	0.279	0.545	0.929	0.279	0.554	0.824	0.278	0.570	0.704
HCP	0.486	1.484	1.847	0.487	1.522	1.846	0.490	1.481	1.810
HCN	0.402	1.186	1.813	0.403	1.236	1.936	0.406	1.200	1.645
OHI	0.671	3.081	5.012	0.671	3.056	5.179	0.669	2.976	4.817
HR	0.503	1.439	1.949	0.501	1.449	1.691	0.502	1.363	1.539
LTC	0.502	2.391	4.420	0.504	2.426	4.625	0.502	2.332	4.114
PLD	0.480	2.151	2.503	0.479	2.154	3.066	0.481	2.049	2.682
EGP	0.349	1.240	2.067	0.350	1.215	2.313	0.350	1.097	1.950
FR	0.632	2.366	3.241	0.632	2.357	3.015	0.637	2.411	3.311
MNRTA	0.325	0.947	1.038	0.327	0.959	1.020	0.327	0.956	0.993
HPT	0.474	1.834	2.007	0.475	1.865	1.878	0.474	1.798	1.944
LHO	0.623	2.212	2.871	0.622	2.236	2.872	0.626	2.265	2.135
FCH	1.090	4.771	8.398	1.089	4.720	8.285	1.086	4.675	7.797
EQR	0.453	1.754	1.482	0.451	1.701	1.583	0.450	1.787	1.806
AVB	0.424	1.348	2.469	0.423	1.367	2.667	0.424	1.396	2.127
ESS	0.413	1.253	2.424	0.416	1.286	2.583	0.417	1.320	2.202
CPT	0.420	1.226	1.634	0.419	1.216	1.895	0.420	1.249	1.488
BRE	0.420	1.355	1.612	0.421	1.349	1.751	0.420	1.434	1.537
HME	0.393	1.230	1.385	0.395	1.251	1.320	0.396	1.324	1.526
AIV	0.541	1.815	1.332	0.543	1.806	1.473	0.543	1.848	1.952
MAA	0.421	1.372	2.179	0.421	1.389	2.344	0.420	1.402	2.143
PPS	0.468	1.982	1.568	0.470	1.984	1.616	0.471	2.124	1.542
CLP	0.382	1.779	1.688	0.382	1.731	1.309	0.384	1.809	1.368
AEC	0.522	2.120	2.711	0.522	2.130	2.723	0.521	2.112	2.681
BXP	0.390	1.352	3.428	0.390	1.387	3.566	0.392	1.325	2.999
SLG	0.568	2.135	3.180	0.569	2.130	3.451	0.571	2.140	3.010
ARE	0.373	1.187	2.788	0.373	1.225	3.078	0.374	1.171	2.832
DRE	0.491	1.950	3.456	0.492	1.906	3.477	0.490	1.806	3.072
CLI	0.394	1.483	2.249	0.393	1.434	2.013	0.391	1.320	1.687
KRC	0.466	1.701	2.554	0.465	1.691	2.837	0.466	1.604	2.605
HIW	0.519	1.142	1.377	0.519	1.180	1.683	0.518	1.170	1.542
OFC	0.505	1.812	3.253	0.503	1.806	3.358	0.501	1.751	3.147
BDN	0.585	2.455	4.003	0.584	2.354	4.057	0.581	2.310	3.448
PKY	0.638	1.740	3.516	0.638	1.736	3.518	0.636	1.722	3.215
SPG	0.414	1.486	2.811	0.415	1.451	2.982	0.410	1.462	2.432
MAC	0.534	2.035	2.639	0.532	1.996	3.139	0.530	1.889	3.016

Table 9 – Continued

	Estimation method: 2.5 yr rolling OLS			Estimation method: 4 yr rolling OLS			Estimation method: Full sample OLS		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Ticker	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
KIM	0.512	1.747	2.162	0.511	1.762	2.490	0.514	1.707	2.339
FRT	0.385	1.506	2.834	0.387	1.529	3.039	0.388	1.480	2.331
O	0.351	1.270	1.273	0.350	1.269	1.567	0.350	1.313	1.772
TCO	0.452	1.566	2.249	0.452	1.565	2.353	0.452	1.574	2.058
REG	0.388	1.328	2.407	0.388	1.307	3.092	0.390	1.224	3.148
DDR	0.530	2.332	2.254	0.526	2.284	2.435	0.529	2.254	2.268
WRI	0.417	1.306	2.175	0.417	1.286	1.672	0.414	1.168	1.416
SKT	0.387	1.249	1.739	0.389	1.300	1.943	0.391	1.279	1.619
CBL	0.600	2.631	5.163	0.601	2.649	4.651	0.597	2.525	4.286
EQY	0.422	1.379	1.663	0.423	1.342	1.988	0.419	1.206	1.811
ALX	0.617	2.388	3.433	0.618	2.392	3.517	0.618	2.347	3.066
GRT	0.681	3.356	3.793	0.683	3.317	3.141	0.681	3.380	3.119
AKR	0.349	1.516	3.553	0.350	1.549	3.651	0.351	1.492	3.294
BFS	0.465	1.258	2.623	0.469	1.283	2.964	0.471	1.307	2.919
PEI	0.623	2.736	3.427	0.621	2.671	3.269	0.625	2.649	3.472
RPT	0.565	1.717	2.081	0.564	1.742	1.933	0.566	1.658	1.685
OLP	0.393	1.490	1.369	0.393	1.476	1.362	0.394	1.473	1.257
ADC	0.449	1.766	1.922	0.450	1.749	2.165	0.449	1.711	2.304
RPI	0.637	2.767	2.849	0.640	2.736	2.884	0.637	2.652	3.172
SSS	0.391	1.470	1.767	0.391	1.431	2.094	0.391	1.391	1.855
Ave	0.484	1.768	2.553	0.484	1.765	2.629	0.484	1.739	2.439

Notes: Table 9 presents standard deviation of absolute forecast errors using CAPM as cost of equity capital (CoEC) model for our 60-firm sample. We examine 3 estimation methods: 2.5 year rolling window ordinary least squares (OLS), 4 year rolling window OLS and full sample OLS. The estimation period is January 1999 to December 2011. The forecast period analyzed were 1 month forecast, 1 year forecast and 5 year forecast. For each firm, we obtained CAPM factor loadings estimated using 2.5 year rolling, 4 year rolling and full sample OLS. Given the factor loadings, we calculated estimated CAPM CoECs with long run average of market risk premium as factor. Using 1927 to 2011 as our sample, annual long run average of market risk premium is 7.94% and annual long run average of risk free rate is 3.63%. We then obtained monthly forecast errors as the difference between 1-month ahead realized annualized return and our estimated CAPM CoECs. Given the monthly forecast errors, we obtained 1 month, 1 year and 5 years absolute forecast errors. We report the standard deviation of absolute forecast errors for each firm. *Ave* is average of 60 firms.

Table 10 – CAPM mean and standard deviation of absolute forecast errors for our REIT Type portfolios and NAREIT index

Panel A: Mean of absolute forecast errors

	Estimation method: 2.5 yr rolling OLS			Estimation method: 4 yr rolling OLS			Estimation method: Full sample OLS		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Property type	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
Diversified	0.490	1.620	1.765	0.492	1.651	1.780	0.490	1.542	1.755
H. Care	0.612	1.950	3.656	0.614	2.000	4.398	0.610	1.957	5.573
Industrial	0.509	2.076	2.395	0.510	2.168	2.203	0.504	2.036	1.977
Lodging	0.845	3.253	6.932	0.846	3.305	6.724	0.842	3.163	6.429
M-family	0.564	2.614	2.891	0.566	2.668	3.460	0.562	2.575	3.872
Office	0.617	2.467	3.348	0.619	2.524	3.082	0.612	2.345	2.376
Retail	0.550	2.519	2.154	0.552	2.588	2.185	0.545	2.439	2.180
S. Storage	0.602	2.547	1.808	0.603	2.593	2.305	0.598	2.449	2.740
ALL	0.535	2.346	1.968	0.537	2.414	2.012	0.531	2.268	2.048
NAREIT	0.648	2.794	4.338	0.651	2.890	3.976	0.646	2.745	3.120

Table 10 – Continued

Panel B: Standard deviation of absolute forecast errors

Property type	Estimation method: 2.5 yr rolling OLS			Estimation method: 4 yr rolling OLS			Estimation method: Full sample OLS		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
Diversified	0.355	0.912	1.176	0.355	0.933	1.375	0.354	0.877	1.309
H. Care	0.427	1.804	2.769	0.427	1.850	3.036	0.430	1.793	2.648
Industrial	0.376	1.280	1.504	0.376	1.247	1.799	0.379	1.229	1.515
Lodging	0.634	2.499	2.907	0.633	2.491	2.787	0.633	2.483	2.729
M-family	0.379	1.262	1.614	0.379	1.245	1.776	0.382	1.331	1.662
Office	0.422	1.339	1.749	0.421	1.325	2.084	0.421	1.223	1.857
Retail	0.351	1.406	2.006	0.351	1.373	2.437	0.353	1.308	2.257
S. Storage	0.391	1.470	1.767	0.391	1.431	2.094	0.391	1.391	1.855
ALL	0.361	1.207	1.820	0.360	1.183	2.232	0.361	1.150	1.985
NAREIT	0.666	2.108	1.548	0.665	2.067	1.720	0.667	2.085	1.803

Notes: Table 10 presents mean and standard deviation of absolute forecast errors using CAPM as cost of equity capital (CoEC) model for our portfolios. We obtain equal-weighted portfolios grouped by property types (REIT Type portfolios). In addition, we obtain equal-weighted portfolios consisting of all 60 firms in our sample (ALL portfolio). Finally, we also report results from NAREIT Equity REITs index. We examine 3 estimation methods: 2.5 year rolling window ordinary least squares (OLS), 4 year rolling window OLS and full sample OLS. The estimation period is January 1999 to December 2011. The forecast period analyzed were 1 month forecast, 1 year forecast and 5 year forecast. For each firm, we obtained CAPM factor loadings estimated using 2.5 year rolling, 4 year rolling and full sample OLS. Given the factor loadings, we calculated estimated CAPM CoECs with long run average of market risk premium as factor. Using 1927 to 2011 as our sample, annual long run average of market risk premium is 7.94% and annual long run average of risk free rate is 3.63%. We then obtained monthly forecast errors as the difference between 1-month ahead realized annualized return and our estimated CAPM CoECs. Given the monthly forecast errors, we obtained 1 month, 1 year and 5 years absolute forecast errors. We report the mean and standard deviation of absolute forecast errors for each portfolio.

Table 11 – Fama and French (1993) three-factor model mean absolute forecast errors for our 60-firm sample.

	Estimation method: 2.5 yr rolling OLS			Estimation method: 4 yr rolling OLS			Estimation method: Full sample OLS		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Ticker	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
EPR	0.660	2.401	4.574	0.663	2.459	5.469	0.659	2.337	4.624
WRE	0.542	1.615	1.556	0.541	1.500	1.503	0.540	1.478	1.324
LXP	0.643	2.275	3.481	0.642	2.215	2.881	0.635	2.178	3.972
CUZ	0.729	2.553	7.568	0.727	2.471	7.385	0.722	2.304	7.520
IRETS	0.438	0.823	2.228	0.439	0.806	1.595	0.440	0.767	0.946
HCP	0.649	1.783	5.426	0.648	1.782	5.768	0.648	1.716	4.985
HCN	0.571	1.562	4.075	0.570	1.594	4.706	0.567	1.480	4.404
OHI	0.796	2.437	3.726	0.800	2.504	3.568	0.805	2.542	5.054
HR	0.666	1.956	3.628	0.666	1.888	3.317	0.666	1.925	3.743
LTC	0.679	2.194	4.727	0.686	2.229	4.666	0.684	2.175	4.635
PLD	0.708	2.687	3.906	0.711	2.741	3.754	0.703	2.535	4.329
EGP	0.553	1.685	2.384	0.553	1.751	2.490	0.551	1.643	2.252
FR	0.811	2.962	6.304	0.813	3.034	6.412	0.812	3.038	8.095
MNRTA	0.401	1.101	1.465	0.400	1.087	1.342	0.399	1.070	1.367
HPT	0.675	2.259	3.246	0.676	2.272	3.572	0.676	2.258	4.270
LHO	0.872	3.474	2.573	0.865	3.267	2.329	0.862	3.267	3.260
FCH	1.308	4.752	18.790	1.300	4.505	19.846	1.306	4.682	22.379
EQR	0.625	2.651	2.574	0.619	2.448	2.649	0.620	2.434	2.111
AVB	0.645	3.105	3.512	0.640	2.940	3.922	0.637	2.839	3.644
ESS	0.617	2.766	3.298	0.610	2.559	3.534	0.612	2.466	2.304
CPT	0.666	2.652	1.537	0.664	2.572	1.515	0.660	2.478	1.359
BRE	0.611	2.375	1.892	0.609	2.266	2.348	0.607	2.117	1.854
HME	0.605	1.711	3.674	0.603	1.626	4.058	0.605	1.623	3.368
AIV	0.778	2.818	2.076	0.778	2.730	1.788	0.780	2.680	2.421
MAA	0.606	2.327	3.277	0.605	2.330	3.312	0.603	2.253	3.257
PPS	0.688	3.294	2.360	0.685	3.178	2.373	0.686	3.073	3.185
CLP	0.567	2.637	2.153	0.567	2.621	2.073	0.566	2.556	2.311
AEC	0.724	2.809	4.289	0.724	2.762	4.294	0.723	2.790	4.652
BXP	0.614	2.916	4.689	0.612	2.814	4.846	0.609	2.712	4.220
SLG	0.799	4.186	3.096	0.800	4.177	3.146	0.792	3.974	2.799
ARE	0.584	1.751	2.037	0.586	1.833	2.106	0.584	1.729	2.347
DRE	0.701	2.512	8.480	0.698	2.418	7.993	0.698	2.342	9.115
CLI	0.669	1.889	5.692	0.668	1.857	5.197	0.666	1.789	5.328
KRC	0.700	3.072	2.846	0.697	3.012	2.805	0.697	2.988	3.180
HIW	0.680	1.718	1.652	0.681	1.769	1.634	0.678	1.623	1.366
OFC	0.665	2.893	4.528	0.666	2.889	4.582	0.656	2.657	4.043
BDN	0.789	2.791	12.470	0.790	2.741	11.589	0.787	2.621	12.179
PKY	0.789	2.484	8.683	0.789	2.461	8.628	0.793	2.487	10.287
SPG	0.647	2.765	4.100	0.648	2.771	4.802	0.644	2.580	3.984
MAC	0.731	3.138	3.400	0.730	3.097	3.197	0.725	2.940	3.586

Table 11 – Continued

	Estimation method: 2.5 yr rolling OLS			Estimation method: 4 yr rolling OLS			Estimation method: Full sample OLS		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Ticker	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
KIM	0.718	3.052	5.512	0.719	3.105	5.365	0.706	2.826	5.461
FRT	0.602	2.382	6.630	0.603	2.368	6.893	0.597	2.171	5.832
O	0.505	1.787	1.804	0.507	1.833	2.242	0.504	1.735	1.972
TCO	0.666	3.115	8.125	0.665	3.063	8.083	0.659	2.895	6.663
REG	0.652	2.446	3.831	0.653	2.472	3.731	0.643	2.173	3.304
DDR	0.820	3.606	3.669	0.819	3.563	3.391	0.807	3.291	4.324
WRI	0.663	2.321	5.294	0.664	2.325	4.623	0.658	2.052	4.455
SKT	0.535	2.075	6.742	0.535	2.070	7.127	0.534	1.990	7.324
CBL	0.880	3.886	12.470	0.881	3.886	11.957	0.877	3.758	12.725
EQY	0.622	2.058	3.139	0.620	2.063	2.848	0.615	1.839	2.791
ALX	0.804	3.114	5.351	0.808	3.141	6.657	0.803	3.056	7.500
GRT	0.927	3.930	5.193	0.924	3.897	4.769	0.923	3.857	6.263
AKR	0.574	2.212	5.515	0.573	2.180	5.009	0.569	2.097	4.376
BFS	0.668	2.146	3.946	0.669	2.175	3.924	0.670	2.147	3.443
PEI	0.885	3.345	7.778	0.884	3.343	7.287	0.874	3.249	9.607
RPT	0.731	2.358	5.386	0.735	2.451	4.275	0.721	2.157	3.855
OLP	0.540	1.873	1.763	0.541	1.941	1.529	0.536	1.776	1.429
ADC	0.628	2.356	2.772	0.629	2.423	2.785	0.624	2.305	2.461
RPI	0.686	2.811	8.024	0.681	2.825	8.237	0.686	2.994	9.224
SSS	0.598	2.416	1.782	0.598	2.446	1.909	0.592	2.263	1.519
Ave	0.682	2.551	4.612	0.681	2.526	4.594	0.678	2.430	4.743

Notes: Table 11 presents mean absolute forecast errors using Fama and French (1993) three-factor model as cost of equity capital (CoEC) model for our 60-firm sample. We examine 3 estimation methods: 2.5 year rolling window ordinary least squares (OLS), 4 year rolling window OLS and full sample OLS. The estimation period is January 1999 to December 2011. The forecast period analyzed were 1 month forecast, 1 year forecast and 5 year forecast. For each firm, we obtained Fama and French (1993) factor loadings estimated using 2.5 year rolling, 4 year rolling and full sample OLS. Given the factor loadings, we calculated estimated Fama and French (1993) three-factor CoECs with long run average of market risk premium, SMB and HML as factor. Using 1927 to 2011 as our sample, annual long run average of market risk premium is 7.94%, annual long run average of SMB is 3.66%, annual long run average of HML is 4.74% and annual long run average of risk free rate is 3.63%. We then obtained monthly forecast errors as the difference between 1-month ahead realized annualized return and our estimated Fama and French (1993) CoECs. Given the monthly forecast errors, we obtained 1 month, 1 year and 5 years absolute forecast errors. We report the mean of absolute forecast errors for each firm. *Ave* is average of 60 firms.

Table 12 – Fama and French (1993) three-factor model standard deviation of absolute forecast errors for our 60-firm sample.

	Estimation method: 2.5 yr rolling OLS			Estimation method: 4 yr rolling OLS			Estimation method: Full sample OLS		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Ticker	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
EPR	0.479	1.621	2.447	0.480	1.667	2.483	0.479	1.574	2.237
WRE	0.380	1.063	1.211	0.378	1.002	1.088	0.382	1.025	1.086
LXP	0.504	1.474	2.555	0.502	1.451	2.198	0.508	1.488	2.507
CUZ	0.512	1.699	3.978	0.510	1.637	3.918	0.511	1.606	4.203
IRETS	0.280	0.589	1.136	0.279	0.590	1.126	0.278	0.580	0.856
HCP	0.486	1.436	1.874	0.485	1.436	1.752	0.484	1.358	1.772
HCN	0.403	1.175	1.913	0.401	1.173	1.942	0.401	1.067	1.645
OHI	0.672	2.692	3.814	0.669	2.708	3.976	0.670	2.805	4.496
HR	0.503	1.347	2.138	0.501	1.331	2.037	0.503	1.307	2.111
LTC	0.507	2.197	4.111	0.501	2.301	4.310	0.500	2.206	4.114
PLD	0.480	2.056	2.691	0.479	2.069	2.931	0.487	2.130	2.015
EGP	0.348	1.116	1.845	0.350	1.103	1.886	0.349	1.040	1.678
FR	0.639	2.401	3.728	0.638	2.405	3.740	0.643	2.543	3.949
MNRTA	0.327	0.925	1.082	0.328	0.957	1.105	0.328	0.920	1.065
HPT	0.479	1.857	2.299	0.477	1.845	2.332	0.475	1.853	2.301
LHO	0.622	2.138	2.428	0.623	2.174	2.028	0.630	2.311	1.783
FCH	1.104	5.169	8.392	1.098	5.076	7.897	1.104	5.295	8.161
EQR	0.453	1.687	1.412	0.451	1.675	1.408	0.453	1.746	1.070
AVB	0.428	1.298	2.607	0.424	1.262	2.502	0.425	1.250	2.127
ESS	0.425	1.397	2.674	0.422	1.347	2.488	0.417	1.307	2.068
CPT	0.422	1.283	1.547	0.421	1.236	1.484	0.423	1.255	1.275
BRE	0.420	1.409	1.435	0.416	1.371	1.620	0.415	1.390	1.233
HME	0.393	1.258	1.370	0.392	1.205	1.349	0.391	1.180	1.454
AIV	0.552	1.804	1.629	0.547	1.761	1.304	0.544	1.792	1.528
MAA	0.422	1.333	2.218	0.421	1.331	2.249	0.421	1.315	2.037
PPS	0.475	2.135	1.821	0.471	2.111	1.835	0.470	2.232	2.371
CLP	0.382	1.847	2.135	0.380	1.791	2.092	0.380	1.828	2.300
AEC	0.515	1.968	2.339	0.514	1.956	2.347	0.515	1.946	2.508
BXP	0.395	1.365	3.312	0.391	1.333	3.238	0.391	1.272	2.924
SLG	0.577	2.303	2.705	0.571	2.198	2.807	0.573	2.195	2.215
ARE	0.374	1.100	1.876	0.374	1.102	2.261	0.372	1.042	2.456
DRE	0.497	2.124	3.800	0.495	2.007	3.616	0.493	2.026	3.121
CLI	0.395	1.496	2.803	0.392	1.410	2.555	0.394	1.375	2.642
KRC	0.469	1.650	2.345	0.467	1.597	2.223	0.468	1.642	1.985
HIW	0.523	1.202	1.431	0.523	1.200	1.325	0.522	1.134	1.068
OFC	0.507	1.690	3.626	0.504	1.691	3.635	0.507	1.625	3.258
BDN	0.606	2.814	4.095	0.599	2.666	4.111	0.598	2.630	3.448
PKY	0.649	1.919	3.915	0.647	1.886	3.851	0.641	1.894	3.215
SPG	0.415	1.368	2.824	0.415	1.344	2.806	0.413	1.337	2.333
MAC	0.537	1.971	2.367	0.535	1.926	2.436	0.537	1.966	1.841

Table 12 – Continued

	Estimation method: 2.5 yr rolling OLS			Estimation method: 4 yr rolling OLS			Estimation method: Full sample OLS		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Ticker	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
KIM	0.512	1.855	2.354	0.512	1.832	2.490	0.519	1.871	2.110
FRT	0.384	1.412	2.967	0.383	1.415	2.933	0.381	1.294	2.331
O	0.351	1.179	1.146	0.351	1.172	1.366	0.354	1.195	1.230
TCO	0.454	1.498	2.297	0.452	1.488	2.186	0.453	1.471	2.058
REG	0.390	1.297	2.660	0.390	1.284	3.045	0.392	1.156	2.059
DDR	0.534	2.385	2.279	0.531	2.332	2.282	0.538	2.384	2.303
WRI	0.424	1.429	2.642	0.423	1.361	2.121	0.421	1.275	2.384
SKT	0.388	1.200	1.774	0.387	1.212	1.857	0.388	1.181	1.619
CBL	0.614	2.816	5.477	0.615	2.866	5.511	0.617	2.877	5.224
EQY	0.424	1.408	1.665	0.426	1.364	1.733	0.425	1.281	1.463
ALX	0.613	2.206	3.558	0.614	2.300	3.684	0.613	2.195	3.066
GRT	0.687	3.368	4.306	0.687	3.307	3.978	0.687	3.351	4.895
AKR	0.346	1.410	3.458	0.347	1.449	3.498	0.349	1.391	3.290
BFS	0.465	1.233	2.723	0.466	1.243	2.679	0.466	1.229	2.531
PEI	0.623	2.755	3.475	0.623	2.713	3.394	0.633	2.814	3.519
RPT	0.570	1.878	2.549	0.569	1.893	2.443	0.571	1.759	2.218
OLP	0.394	1.516	1.769	0.393	1.485	1.465	0.393	1.451	1.531
ADC	0.453	1.728	1.869	0.452	1.704	1.857	0.451	1.636	1.743
RPI	0.635	2.766	3.130	0.637	2.714	3.211	0.633	2.625	3.923
SSS	0.390	1.409	1.778	0.390	1.383	1.857	0.392	1.382	1.366
Ave	0.487	1.769	2.630	0.485	1.747	2.598	0.486	1.738	2.455

Notes: Table 12 presents standard deviation of absolute forecast errors using Fama and French (1993) three-factor model as cost of equity capital (CoEC) model for our 60-firm sample. We examine 3 estimation methods: 2.5 year rolling window ordinary least squares (OLS), 4 year rolling window OLS and full sample OLS. The estimation period is January 1999 to December 2011. The forecast period analyzed were 1 month forecast, 1 year forecast and 5 year forecast. For each firm, we obtained Fama and French (1993) factor loadings estimated using 2.5 year rolling, 4 year rolling and full sample OLS. Given the factor loadings, we calculated estimated Fama and French (1993) three-factor CoECs with long run average of market risk premium, SMB and HML as factor. Using 1927 to 2011 as our sample, annual long run average of market risk premium is 7.94%, annual long run average of SMB is 3.66%, annual long run average of HML is 4.74% and annual long run average of risk free rate is 3.63%. We then obtained monthly forecast errors as the difference between 1-month ahead realized annualized return and our estimated Fama and French (1993) CoECs. Given the monthly forecast errors, we obtained 1 month, 1 year and 5 years absolute forecast errors. We report the standard deviation of absolute forecast errors for each firm. *Ave* is average of 60 firms.

Table 13 – Fama and French (1993) three-factor mean and standard deviation of absolute forecast errors for our REIT Type portfolios and NAREIT index

Panel A: Mean of absolute forecast errors

	Estimation method: 2.5 yr rolling OLS			Estimation method: 4 yr rolling OLS			Estimation method: Full sample OLS		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Property type	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
Diversified	0.485	1.586	2.383	0.486	1.548	2.040	0.484	1.428	1.953
H. Care	0.603	1.796	3.000	0.606	1.820	3.146	0.605	1.779	3.180
Industrial	0.501	1.905	2.631	0.503	1.981	2.645	0.499	1.875	2.998
Lodging	0.844	3.115	7.395	0.839	2.962	7.871	0.839	3.020	9.602
M-family	0.559	2.541	1.988	0.556	2.431	2.141	0.554	2.339	1.833
Office	0.609	2.351	4.210	0.609	2.330	3.976	0.603	2.172	3.929
Retail	0.543	2.436	2.470	0.544	2.449	2.289	0.536	2.269	2.287
S. Storage	0.598	2.416	1.782	0.598	2.446	1.909	0.592	2.263	1.519
ALL	0.526	2.247	2.341	0.526	2.225	2.189	0.522	2.086	2.246
NAREIT	0.637	2.646	5.332	0.639	2.650	5.016	0.633	2.512	5.548

Table 13 – Continued

Panel B: Standard deviation of absolute forecast errors

Property type	Estimation method: 2.5 yr rolling OLS			Estimation method: 4 yr rolling OLS			Estimation method: Full sample OLS		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
Diversified	0.362	0.926	1.313	0.359	0.898	1.295	0.359	0.868	1.365
H. Care	0.430	1.672	2.524	0.426	1.667	2.607	0.427	1.608	2.454
Industrial	0.379	1.290	1.337	0.379	1.278	1.357	0.382	1.320	1.460
Lodging	0.641	2.681	3.462	0.638	2.650	3.632	0.643	2.826	4.127
M-family	0.384	1.296	1.456	0.382	1.260	1.483	0.384	1.320	1.095
Office	0.433	1.488	1.943	0.429	1.427	1.887	0.432	1.412	1.963
Retail	0.356	1.370	1.765	0.355	1.343	1.988	0.358	1.299	1.440
S. Storage	0.390	1.409	1.778	0.390	1.383	1.857	0.392	1.382	1.366
ALL	0.370	1.207	1.498	0.367	1.172	1.599	0.369	1.178	1.289
NAREIT	0.675	2.282	1.975	0.672	2.212	1.831	0.678	2.348	2.891

Notes: Table 13 presents mean and standard deviation of absolute forecast errors using Fama and French (1993) three factor model as cost of equity capital (CoEC) model for our portfolios. We obtain equal-weighted portfolios grouped by REIT Types (REIT Types portfolios). In addition, we obtain equal-weighted portfolios consisting of all 60 firms in our sample (ALL portfolio). Finally, we also report results from NAREIT Equity REITs index. We examine 3 estimation methods: 2.5 year rolling window ordinary least squares (OLS), 4 year rolling window OLS and full sample OLS. The estimation period is January 1999 to December 2011. The forecast period analyzed were 1 month forecast, 1 year forecast and 5 year forecast. For each firm, we obtained Fama and French (1993) factor loadings estimated using 2.5 year rolling, 4 year rolling and full sample OLS. Given the factor loadings, we calculated estimated Fama and French (1993) three-factor CoECs with long run average of market risk premium as factor. Using 1927 to 2011 as our sample, annual long run average of market risk premium is 7.94% and annual long run average of risk free rate is 3.63%. We then obtained monthly forecast errors as the difference between 1-month ahead realized annualized return and our estimated Fama and French (1993) three-factor CoECs. Given the monthly forecast errors, we obtained 1 month, 1 year and 5 years absolute forecast errors. We report the mean and standard deviation of absolute forecast errors for each portfolio.

Figure 1 – CUSUM plots for the temporal stability of CAPM beta.

Panel A: CUSUM plots of CAPM for firm OHI, LTC, FRT, TCO, DDR and ALX

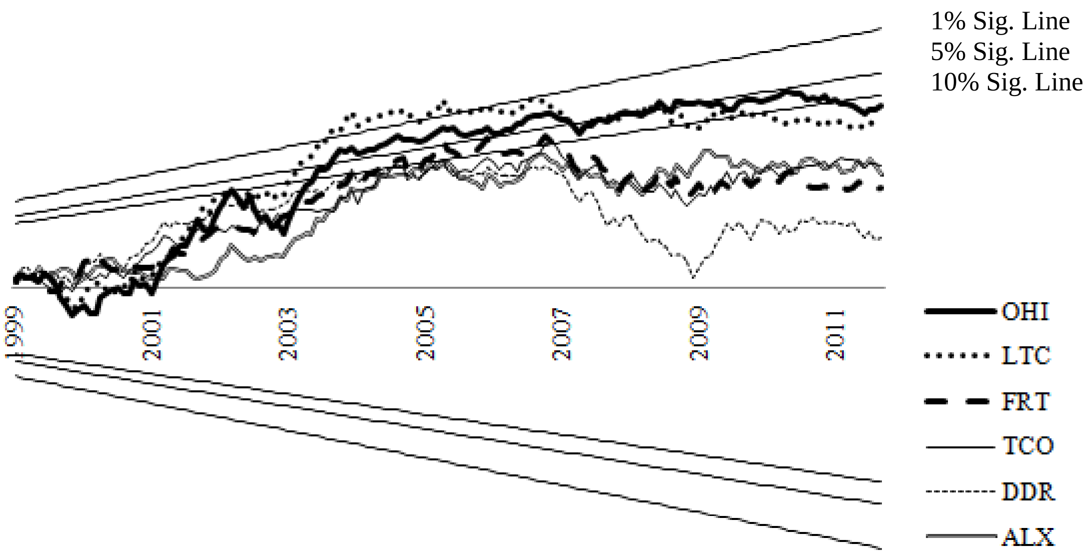
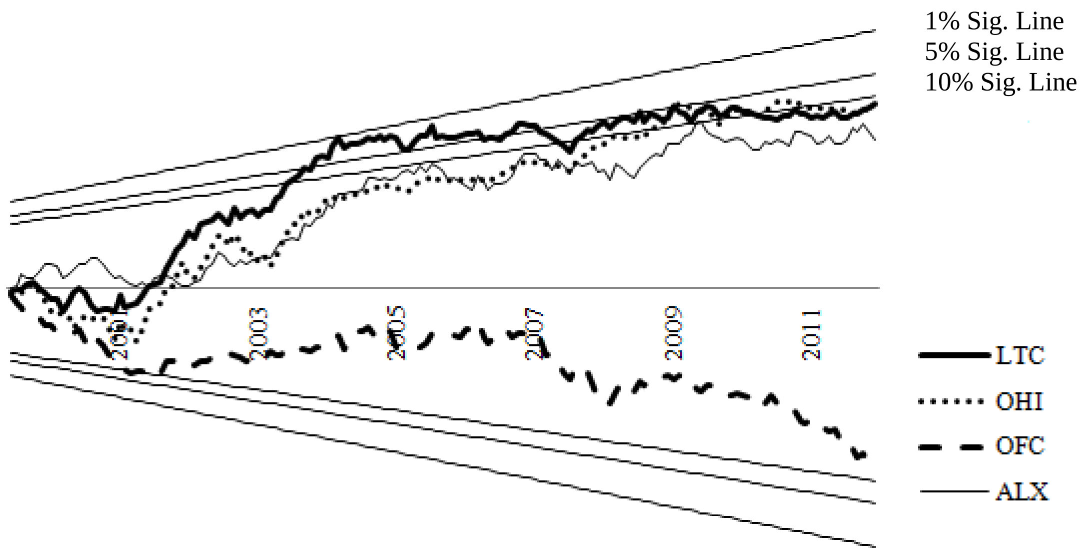


Figure 1 – Continued

Panel B: CUSUM plots for the three-factor model for firm LTC, OHI, OFC and ALX



Notes: Figure 1 only reports CUSUM plots of firms where the null is rejected. The null of constant factor loading under CAPM is shown in (5) while that of Fama and French (1993) three-factor model is shown in (8). For each firm, we obtain loadings using ordinary least squares and calculate recursive residuals from (6) as

$$w_t = \frac{y_t - x_t^{'} b_{t-1}}{\sqrt{1 + x_t^{'} \left(X_{t-1}^{'} X_{t-1} \right)^{-1} x_t}}$$

where *y* is the dependent variable, *x* is the independent variables, and *b* is the estimated loading. We then obtained CUSUM as

$$W_t = \frac{1}{s} \sum_{j=k+1}^{t} w_j$$

where *s* is the recursive residual's sample standard deviation. The period is January 1999 to December 2011. *Sig. Line* refers to statistical significance line. We plot 1%, 5% and 10% significance lines respectively. When a firm's CUSUM plot crosses an upper or lower statistical significance line, the null hypothesis of constant factor loadings for that particular firm is rejected.

CHAPTER 2

On the Cost of Equity Capital of Real Estate Investment Trusts (REITs): Estimation and Predictability of Time-varying Market Risk Factor Loading

2.1 Introduction

An important concept in financial economics, cost of equity capital (CoEC) is useful in capital budgeting and critical in valuations of cash flows. A number of studies have investigated time variation in risk characteristic of Equity REITs. An early study by Khoo, Hartzell, and Hoesli (1993) conclude that the risk loadings for Equity REITs in the 1980s were significantly lower than in the 1970s. Chiang, Lee and Wisen (2005) report that that betas decline from 1972 to 2002. Khoo, Hartzell and Hoesli (1993) use 14 individual REITs data while Chiang, Lee and Wisen (2005) use NAREIT index data. Liang, McIntosh and Webb (1995) however find that the null of constant beta cannot be rejected and Arifin (2013) finds inconclusive evidence regarding inter-temporal stability of risk factor loadings. Using a richer econometric toolset, the latest study by Case, Guidolin and Yildirim (2013) document the presence of a Markov switching regime in National Association of Real Estate Investment Trusts (NAREIT) Equity REITs index' expected return and variance.

In this study, we contribute to the discourse by investigating models with explicit dynamic of market risk characteristics. Zhou (2013) compares out-of-sample performance of several specifications of CAPM with dynamic beta. Using NAREIT Equity REITs index data from 1999 to 2011, Zhou (2013) finds that state space model with random walk CAPM beta obtains the best forecast performance. In this study, we consider market risk premium characteristic as an autoregressive process of order 1 in addition to random walk. Technically, given the lack of mean reversion, a random walk process allows an explosive market risk beta – a behavior not observed in the financial market. We cast our model as a space state specification within the context of a Bayesian framework – a modeling approach supported by Zhou (2013) result.

In addition to Sharpe (1964) single-factor CAPM studied by Zhou (2013), we also study market risk premium dynamic within the context of Fama and French (1993) three-factor model. Over and above the empirical support for size and book-to-market factors in the literature, Maio and Santa-Clara (2012) find evidence that Fama and French (1993) satisfy the restrictions of Merton (1973) Intertemporal CAPM specification – a distinction shared only with Carhart (1997) model within the 8 multifactor models considered by Maio et al (2012). In our formulation, we specify that SMB and HML loadings are time invariant. Arifin (2013) suggests that market risk premium loading may vary over time under both single-factor and three-factor CoEC models. At the same time however, Arifin (2013) suggests that SMB and HML loadings vary less over time. We consider the notion suggested by Arifin (2013) and we model market risk premium loading as time varying while assuming a time invariant SMB and HML loading. Sixty Equity REITs are considered in our study, in contrast to index data investigated by Zhou (2013). A recent study by Ooi, Wang and Webb (2009) find that REITs idiosyncratic risk renders market risk premium, size and book-to-market statistically unimportant at the firm level. As per MacKinlay (1995), idiosyncratic risk serves as a proxy for an exposure to a missing risk factor in pricing model with non-zero pricing error (i.e. non-zero regression intercept) – Arifin (2013) finds evidence of non-zero pricing error for a large proportion of our 60-firm sample. Given the lack of identification for the missing risk factor, we defer the evidence of Ooi et al (2009) and utilize commonly used systematic risk factors: market risk premium, size and book-to-market in our present investigation.

Our empirical analysis focuses on CoEC forecast. In the context of valuation, a manager discounting near term cash flow may prefer a recent discount rate that reflect the latest market condition. The choice of discount rate to value more distant cash flows however may depend on the stochastic process followed by the discount rate. Our results are as follows. Using 60 Equity REITs firm data, firm-level results from single-factor CAPM show that about half of our sample obtains improved distant out-of-sample accuracy under both random walk process and autoregressive of order 1 process. Among those firms that show improvements through explicit modeling of single-factor beta dynamics however, random walk process appears to obtain a more accurate distant forecast. We also find at the same time that the standard deviation of distant forecast error increases when we explicitly model the time variation of market risk loading. In choosing distant discount rate to value distant cash flows therefore, a manager has to balance between obtaining a lower mean absolute forecast error and a lower standard deviation of absolute forecast error. Finally, the other half of our sample appears to have time-invariant beta even for distant forecast. Firm-level results from three-factor CoEC model show that about half of our sample also obtains improved distant forecast accuracy when we explicitly modeled market risk loading dynamics over time. Again, random walk appears to be the stochastic process that obtains smaller distant forecast error. Furthermore, firms obtaining an improved out-of-sample fit do not appear to obtain inferior forecast error dispersion based on the three-factor model.

We present our data next. Discussions of time-varying single-factor and three-factor CoEC models follow. We then present empirical results and subsequently conclude.

2.2 Data

We obtained a list of Equity REITs from SNL and National Association of Real Estate Investment Trusts (NAREIT) as of December 2011 as our data. We require that each firm has at least 10 years of operating history. Monthly return data for each firm were downloaded from CRSP. We obtained 60 firms as our complete sample consisting of Diversified, Health Care, Industrial, Lodging, Multi-family, Office, Retail and Self-storage property type. We chose January 1999 as our start date. While we have firm-level data prior to January 1999, keeping a common sample will eliminate cross-section variations among firms due to differing sample periods. Our sample period is then January 1999 to December 2011.

Ticker, firm name and each firm's property type are presented in Table 1. In addition, Table 1 reports market value, annualized firm returns – average, standard error, skewness and kurtosis – and monthly trading volume – average and standard deviation. We obtain return and trading data from CRSP. We annualize by multiplying with 12. Our sample contains a wide range of market valuation, ranging from $13 million for Roberts Realty Investors (RPI) to $38 billion for Simon Property Group (SPG). The average return of our 60-firm sample is 0.16 with standard error of

0.09. Standard error is obtained as standard deviation divided by the square root of the number of data. At the firm-level, sample mean can be further away from the population mean. For example, RPI reports a standard error of 0.106 with an average of 0.051. Table 1 also shows that our sample is non-normal with negative skewness for 34 firms out of 60 and kurtosis larger than 3 for all 60 firms. To minimize the effect of outliers, firm returns are therefore winsorized at 5% and 95%. Finally, Table 1 shows that our sample is actively traded – an average 4 million shares are traded every month. Trading vary widely however with standard deviation of 9.68 million per month.

We also need factor data. The risk-free rate, SMB and HML data are obtained from Dr. Kenneth French's website while value-weighted CRSP index returns surrogate for the market portfolio return. As factors, we use long run average from 1927 to 2011. In using a longer period, we follow Fama and French (1997) example. We hope to better capture business cycle and obtain a more accurate risk factors estimate by using a longer sample period. We present our factor summary statistics in Table 2. The risk free rate averages 3.63% per year. On the other hand, market risk premium $R_{m,t} - R_{f,t-1}$ averages 7.94% per year with a standard error of 2.3% per year – the standard deviation is more volatile at 20.8% per year. We also report the size (SMB) and book-to-market (HML) factor. The average annual SMB factor is 3.66% with a standard deviation of 14.2% per year and a standard error of 1.5% per year. The annual HML factor is 4.73% with standard deviation of 13.9% per year and a standard error of 1.5% per year. The size and book-to-market factor are therefore also volatile in the 1927 to 2011 sample.

2.3 Models of Time-varying Cost of Equity Capital

This section discusses the Markov Chain Monte Carlo (MCMC) CoEC models analyzed in this study. We consider single-factor CAPM CoEC of Sharpe (1964) and three-factor CoEC of Fama and French (1993). We explicitly model the dynamic of CoEC loadings over time. A number of studies have suggested that Equity REITs risk characteristic may vary over time. The latest by Arifin (2013) suggests that market risk premium loading may vary over time in a single-factor CoEC. At the same time, within a Fama and French (1993) three-factor context, Arifin (2013) suggests that there may be less variation over time in the SMB and HML loadings compared to market risk premium loading. In this study, we explicitly model the dynamic of market risk loading as a random walk (RW) and first order autoregressive (AR(1)) process. In the context of Fama and French (1993) model, we assume that SMB and HML loadings are constant.

The benchmark models considered are MCMC for CAPM CoEC with constant beta and MCMC for Fama and French (1993) three-factor model with constant loadings. The constant loadings are modeled with uninformative prior. These benchmarks correspond to the commonly used CoEC estimated via full sample ordinary least squares (OLS). The alternative models are MCMC for CAPM with RW and AR(1) beta as well as MCMC for Fama and French (1993) three-factor model with RW and AR(1) market risk premium loadings. We start by discussing MCMC for single-factor CoEC. We then discuss MCMC for three-factor Fama and French (1993) CoEC.

2.3.1 MCMC for single-factor cost of equity capital

We first consider the benchmark single factor model: MCMC for CAPM CoEC with constant beta. The *ex-post* single factor CAPM is:

$$R_t - R_{f,t-1} = \alpha + \beta(R_{m,t} - R_{f,t-1}) + \varepsilon_t \tag{1}$$

The idiosyncratic returns ε_t are modeled as a sequence of identically, independently distributed (*iid*) normal variables with mean zero and variance σ_ε^2. For each firm, equation (1) contains only three unknown parameters: $\Theta = \left((\alpha, \sigma_\varepsilon^2), \beta\right)$.

To develop the joint prior distribution we assume vague (i.e., uninformative) parameters. The pricing errors α have prior normal distribution with mean 0 and variance 10^2. For the market risk β we assume a normal density again with 0 mean and variance 10^2. The prior for the residual return variance is inverted gamma (IG) with shape and scale parameters $C_1 = C_2 = 0.01$. Thus, the prior joint distribution for the model parameters is given by the product:

$$\pi(\Theta) = N\left(0, 10^2\right) \times N\left(0, 10^2\right) \times IG\left(0.01, 0.01\right) \tag{2}$$

For this simple model, the full conditional distribution is easily derived. The first parameter block is for the CAPM pricing error and the variance of the residual returns: $(\alpha, \sigma_\varepsilon^2)$. To obtain the likelihood function, conditional on beta, define the unexpected return variable: $y_t = (R_t - R_{f,t-1}) - \beta(R_{m,t} - R_{f,t-1})$ for time periods t=1, … , T. It is clear from the CAPM assumptions in equation (1) that y_t is an *iid* sequence of normal random variables with mean α, and variance σ_ε^2. Therefore, the likelihood function for the sample $Y = (y_1, y_2, \ldots, y_T)$, expressed as a function of $(\alpha, \sigma_\varepsilon^2)$ only, is given by:

$$L(\alpha,\sigma_{\varepsilon}^{2}) = (2\pi\,\sigma_{\varepsilon}^{2})^{-T/2}\exp\left[-\frac{1}{2\sigma_{\varepsilon}^{2}}\sum_{t=1}^{T}(y_t-\alpha)^2\right] \tag{3}$$

If we use Bayes' theorem, then we obtain a normal full conditional posterior distribution for the pricing error α (given the data and the remaining parameters) with mean and variance:

$$E\alpha = \left(\frac{T}{\sigma_{\varepsilon}^{2}}+\frac{1}{10^2}\right)^{-1}\left[\frac{T\ \bar{y}}{\sigma_{\varepsilon}^{2}}\right] \text{ and } V\alpha = \left(\frac{T}{\sigma_{\varepsilon}^{2}}+\frac{1}{10^2}\right)^{-1} \tag{4A}$$

where $\bar{y}$ is the sample mean of Y. For the idiosyncratic variance, the likelihood function times the prior IG density yields the following probability: $P(\sigma_{\varepsilon}^{2}) \propto (\sigma_{\varepsilon}^{2})^{-(T/2)}\exp\left[-\frac{1}{2\sigma_{\varepsilon}^{2}}\sum_{t=1}^{T}(y_t-\alpha)^2\right]\frac{\exp\left(-C_2/\sigma_{\varepsilon}^{2}\right)}{(\sigma_{\varepsilon}^{2})^{C_1+1}}$. By inspection, we recognize that the full conditional posterior distribution of σ_{ε}^{2} is inverted gamma:

$$IG(C_1+\frac{T}{2},\ C_2+\frac{1}{2}\sum_{t=1}^{T}(y_t-\alpha)^2) \tag{4B}$$

The second parameter block is for the market factor loading. To analyze β, we define a new excess return variable: $y_t = (R_t - R_{f,t-1}) - \alpha$, for t=1, … , T. Then, the likelihood function for this sample -- expressed as a function of β, is proportional to:

$$L(\beta) \propto (\sigma_\varepsilon^2)^{-T/2} \exp\left[-\frac{1}{2\sigma_\varepsilon^2} \sum_{t=1}^{T} \left(y_t - \beta(R_{m,t} - R_{f,t-1}) \right)^2 \right] \tag{5}$$

Multiplying the prior by this likelihood, we obtain a normal full conditional posterior distribution with mean and variance:

$$E\beta = V\beta \times \left(\frac{1}{\sigma_\varepsilon^2} \sum_{t=1}^{T} y_t (R_{m,t} - R_{f,t-1}) \right) \text{ and } V\beta = \left(\frac{1}{10^2} + \frac{1}{\sigma_\varepsilon^2} \sum_{t=1}^{T} (R_{m,t} - R_{f,t-1})^2 \right)^{-1} \tag{6}$$

Let $P\left((\alpha, \sigma_\varepsilon^2), \beta | Data\right)$ be the joint posterior distribution of the mean, variance, and beta – given the sample of returns data. Advances in Bayesian methods (e.g., Gelfand and Smith, 1990), show that we may simulate the joint posterior by first taking a random sample from the marginal Normal-Inverted Gamma distribution $P\left((\alpha, \sigma_\varepsilon^2) | \beta, Data\right)$, Equations (4A) and (4B). Then, given the pair of values $(\alpha, \sigma_\varepsilon^2)$, we generate a new random draw for the market beta from a normal distribution -- $P\left(\beta | (\alpha, \sigma_\varepsilon^2), Data\right)$, as described in Equation (6).

We use a Monte Carlo Markov Chain (MCMC) to estimate, in turn, the unknown parameters and then the CoEC.[14] The Gibbs sampling algorithm consists of the following steps:

Algorithm 1: CoEC based on the CAPM with constant beta.

1. Draw β from a Normal distribution with mean and variance given by Equation (6).

[14] The literature, both theoretical and empirical, on Bayesian MCMC analysis is vast, Gelfand and Smith (1990) is a seminal reference. See also Chib and Greenberg (1996) for econometrics applications.

2. Using the long run average of the risk free rate $\bar{R}_f$ *= 0.0363 and market risk premium (RP)* $\bar{R}_m - \bar{R}_f$ *= 0.0794, compute the Bayesian CoEC:* $C_{CAPM} = \bar{R}_f + (\bar{R}_m - \bar{R}_f)\beta$

3. Go back to step 1 and repeat K times.

For the initial iteration (k=0) we set the three unknown parameters Θ equal to the full sample OLS estimates for each firm. We generate K=2,000 random draws from the posterior CoEC distribution. To allow proper "mixing" of the conditional distributions (the burn-in period) we discard the first 1,000 observations; also, we keep every fifth observation to minimize the impact of serial correlation in the chain. The remaining 200 observations represent the final sample from the posterior distribution of the CoEC implied by the CAPM with constant market beta. We use the mean of these observations as the final estimate of each parameter and the Bayesian CoEC. The variance across the 200 estimates measures the degree of uncertainty.

An implicit assumption in the CAPM, as defined in Equation (1), is that the market factor loading is time invariant. Several studies have noted that CAPM's beta may vary with time. An early study in the real estate literature by Khoo, Hartzell and Hoesli (1993) provides evidence that beta declines during the 1970s and 1980s. Furthermore, using the NAREIT Equity REITs index from 1972 to 2002, Chiang, Lee and Wisen (2005) present weak evidence of a declining beta. More recently, Arifin (2013) suggests that CAPM's beta might with time using firm data from 1999 to 2011 although as a whole the evidence of time-varying beta is inconclusive. We relax the assumption of a constant market risk, and estimate the CoEC assuming that the time variation in beta is like that of a slow moving RW. Clearly this assumption cannot hold true literally because it allows the possibility of extremely large, or small, betas in the long run. The main advantage of the RW model is that it leads to a parsimonious description for the local behavior of beta and, in turn, time-varying cost of capital estimates.

The conditional *ex-post* single factor CAPM, with a time-varying beta, may be defined as:

$$R_t - R_{f,t-1} = \alpha + \beta_t (R_{m,t} - R_{f,t-1}) + \varepsilon_t \tag{7A}$$

and the factor loading on the market risk factor is modeled as a RW:

$$\beta_t = \beta_{t-1} + \eta_t \tag{7B}$$

where the beta innovation terms η_t are serially independent and normally distributed with mean zero and variance σ_η^2; they are also uncorrelated with the idiosyncratic returns ε_{t+1}. The CAPM with a RW beta requires only three parameters: $\Theta = \left((\alpha,\ \sigma_\varepsilon^2); \sigma_\eta^2\right)$. We base the analysis on a vague prior joint distribution: $\pi(\Theta) = N\left(0,\ 10^2\right) \times IG(C_1, C_2) \times IG(C_1, C_2)$. We set $C_1 = C_2 = 0.01$. From the point of view of classical econometrics, this model appears intractable because beta is not directly observable. But we may employ the Bayesian paradigm to augment the observed returns data with a “pseudo” time series sample of betas as if they were generated by a RW. The procedure to estimate this model involves two steps: First, we develop the Kalman Filter recursions to describe the probability distribution of β_t for each period t. And second, we use a backward recursion algorithm to generate a “pseudo” time series for beta: $\widetilde{B} = (\widetilde{\beta}_1,\ \widetilde{\beta}_2,\ \ldots,\ \widetilde{\beta}_T)$

-- conditional on the data and the model parameters. A time varying cost of capital then follows immediately given the riskless rate and the market risk premium.

Let $\Re_s$ be the sample history of firm and market portfolio returns up to time s. We use $E\beta_{t|s}$ and $V\beta_{t|s}$, respectively, to denote the conditional expected value and variance of β_t in the RW given information up to time s. Using Equation (7B), it is clear that the conditional expectation of next period's beta is equal to the current value $E\beta_{t+1|t} = E\beta_{t|t}$, with a predictive variance equal to $V\beta_{t+1|t} = \sigma_\eta^2 + V\beta_{t|t}$. Next, define the excess return variable: $y_{t+1} = (R_{t+1} - R_{f,t}) - \alpha$; then using information up to time t, we may form next period's expectation as a function of the predicted beta: $E(y_{t+1} \mid \Re_t) = (R_{m,t+1} - R_{f,t})E\beta_{t+1|t}$. The conditional variance is given by: $V(y_{t+1} \mid \Re_t) = (R_{m,t+1} - R_{f,t})^2 V\beta_{t+1|t} + \sigma_\varepsilon^2$, and the conditional covariance with next period's beta is equal to $Cov(y_{t+1}, \beta_{t+1} \mid \Re_t) = (R_{m,t+1} - R_{f,t})V\beta_{t+1|t}$. Last, because the two variables (y_{t+1}, β_{t+1}) are jointly bivariate normal, we may use the partitioning theorem for normal random variables to incorporate the new information observed at time t+1.

Thus, to complete the first step, we note that the conditional distribution of β_{t+1} is normal with updated mean:

$$E\beta_{t+1|t+1} = E\beta_{t+1|t} + K_{t+1}\hat{\varepsilon}_{t+1|t} \tag{8A}$$

where $\hat{\varepsilon}_{t+1|t} = y_{t+1} - (R_{m,t+1} - R_{f,t})E\beta_{t+1|t}$ is defined as the prediction error, and $K_{t+1} = \dfrac{(R_{m,t+1} - R_{f,t})V\beta_{t+1|t}}{\sigma_{\varepsilon}^2 + (R_{m,t+1} - R_{f,t})^2 V\beta_{t+1|t}}$ is the well known Kalman Filter gain. The updated variance is:

$$V\beta_{t+1|t+1} = V\beta_{t+1|t} - \left[(R_{m,t+1} - R_{f,t})V\beta_{t+1|t}\right]K_{t+1} \tag{8B}$$

For the second step, we use a methodology developed by Carter and Kohn (1994) to obtain a "pseudo" time series sample of market factor loadings ($\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_T$) needed to compute the cost of capital for each firm. To show how the process works, we observe that at the end of the sample period T, the distribution of the last beta may be characterized by a normal distribution with well defined mean and variance: $\beta_T \sim N\left(E\beta_{T|T}, V\beta_{T|T}\right)$. These two moments were derived in Equations (10) in the last step of the Kalman Filter. We take a random draw from this distribution and denote its "observed" value as $\tilde{\beta}_T$. Next, we use the RW process (7B) to characterize the distribution of β_T from the point of view of one period earlier. This distribution is normal with conditional moments of $E(\beta_T \mid \Re_{T-1}) = E\beta_{T-1|T-1}$ for the mean, $V(\beta_T \mid \Re_{T-1}) = \sigma_{\eta}^2 + V\beta_{T-1|T-1}$ for the variance, and $Cov(\beta_{T-1}, \beta_T \mid \Re_{T-1}) = V\beta_{T-1|T-1}$ for the covariance. These results may be summarized with a joint bivariate normal distribution for the consecutive pair of betas: $\begin{pmatrix} \beta_{T-1} \\ \beta_T \end{pmatrix} | \Re_{T-1} \sim N\left\{\begin{pmatrix} E\beta_{T-1|T-1} \\ E\beta_{T-1|T-1} \end{pmatrix}, \begin{pmatrix} V\beta_{T-1|T-1} & V\beta_{T-1|T-1} \\ V\beta_{T-1|T-1} & \sigma_{\eta}^2 + V\beta_{T-1|T-1} \end{pmatrix}\right\}$.

Once again, we apply the partitioning theorem for normal random variables to incorporate the new information contained in the "pseudo" value $\tilde{\beta}_T$. The updated mean is

$$E\left(\beta_{T-1} \mid \tilde{\beta}_T, \Re_{T-1}\right) = E\beta_{T-1|T-1} + \frac{V\beta_{T-1|T-1}}{\sigma_\eta^2 + V\beta_{T-1|T-1}}(\tilde{\beta}_T - E\beta_{T-1|T-1}) \tag{9A}$$

and its variance is given by:

$$V\left(\beta_{T-1} \mid \tilde{\beta}_T, \Re_{T-1}\right) = V\beta_{T-1|T-1}\left(1 - \frac{V\beta_{T-1|T-1}}{\sigma_\eta^2 + V\beta_{T-1|T-1}}\right) \tag{9B}$$

Given these moments we may generate $\tilde{\beta}_{T-1}$ as a random draw from a normal distribution with conditional mean (9A) and variance (9B). Then, we may work sequentially backwards to obtain the time sequence of betas: $\tilde{\beta}_{T-2}, \ldots, \tilde{\beta}_1$.

Given this "pseudo" sample, the three unknown parameters in the CAPM with a RW beta model $\Theta = \left((\alpha,\, \sigma_\varepsilon^2); \sigma_\eta^2\right)$ have easily derived posterior distributions. To show this, define the unexpected return variable: $y_t = (R_t - R_{f,t-1}) - \tilde{\beta}_t(R_{m,t} - R_{f,t-1})$ for t=1, … , T. Then, assuming uninformative priors, the full conditional posterior distribution for $(\alpha, \sigma_\varepsilon^2)$ are the same as (4A) and (4B). For the variance of the beta innovations (σ_η^2), conditional on the time sequence of betas, define the time series of filtered observations as: $y_t = \tilde{\beta}_t - \tilde{\beta}_{t-1}$ for t=2, … , T. It is clear from the RW model assumptions that y_t is an *iid* sequence of normal random variables with 0 mean and variance σ_η^2. Thus, the full conditional posterior distribution may be derived quite easily as: $IG(C_1 + \frac{T-1}{2}, C_2 + \frac{1}{2}\sum_{t=2}^{T} y_t^2)$. The MCMC algorithm consists of the following steps:

Algorithm 2: CoEC based on the CAPM with Random Walk beta.

1. Generate a "pseudo" time series beta: $\tilde{B} = (\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_T)$.

2. Using the long run average of the risk free rate $\bar{R}_f$ *= 0.0363 and market risk premium (RP)* $\bar{R}_m - \bar{R}_f$ *= 0.0794, compute the Bayesian CoEC for each period t:* $C_t^{RWCAPM} = \bar{R}_f + (\bar{R}_m - \bar{R}_f)\beta_t$.

3. Go back to step 1 and repeat K times.

As before, for the initial iteration (k=0) we set the three unknown parameters Θ equal to the mean and variance of 36-month rolling OLS estimates. We generate K=2,000 Bayesian CoEC. After 1,000 burn-in estimates, we keep every fifth observation thereafter. The collected 200 observations represent the final sample from the posterior distribution of the CAPM CoEC with RW beta. We use the mean of these observations as the final estimate of each parameter and the CoEC; variability is represented by the variance of the 200 observed parameters and CoEC.

The RW process of the previous section is only one of many potential time series models for factor loadings. Furthermore, it may not be the best description of beta. Technically, a random walk has no fixed mean; because the mean level is constantly changing, it is difficult to estimate such a parameter. Additionally, we know that a random walk lacks mean reversion. Thus, betas may grow or decline indefinitely. Therefore, in this section we consider an AR(1) process to model the behavior of beta. A mean reverting model offers a compromise between the two extremes of a constant market factor loading and a RW process. An AR(1) process may be described as:

$$\beta_t = (1-\varphi)\beta + \varphi\beta_{t-1} + \eta_t \tag{10}$$

where β is the long run (unconditional) mean, and ϕ measures the strength of mean reversion. For example, as ϕ increases from 0 to 1 the strength of mean reversion decreases and betas become less predictable. We make the usual assumptions to insure the process is stationary and mean reverting. The innovation terms η_t are serially independent and normally distributed with mean zero and variance σ_η^2. The idiosyncratic returns ε_{t+1} are modeled in a similar fashion, but with variance σ_ε^2. This model requires five unknown parameters: $\Theta = \left((\alpha,\ \sigma_\varepsilon^2);\ (\phi, \beta, \sigma_\eta^2)\right)$.

To develop the joint prior distribution we proceed as follows. For the pair $\left(\alpha,\ \sigma_\varepsilon^2\right)$ we assume a vague normal-*IG* distribution. The autoregressive parameter ϕ is assumed to follow a truncated normal with zero mean and large variance. To insure that the time series process for the market factor loading is stationary, the support is truncated to the range (-1, 1). For the long run beta β we assume a normal density with zero mean, and large variance. The prior for each variance is inverted gamma with shape and scale parameters $C_1 = C_2 = 0.001$. Thus, the prior joint distribution for the model parameters is given by the product:

$\pi(\Theta) = N\left(0,10^2\right) \times IG(C_1, C_2) \times TN\left(0,10^2\right) \times N\left(0,10^2\right) \times IG(C_1, C_2)$. The Kalman Filter recursions are straightforward. They are the same as in Equations (8) provided we use $E\beta_{t+1|t} = (1-\phi)\beta + \phi E\beta_{t|t}$ to predict next period's beta, and set the prediction variance equal to $V\beta_{t+1|t} = \sigma_\eta^2 + \phi^2 \, V\beta_{t|t}$.

For the second step in generating the backward recursions for market factor loading $\tilde{\beta}_{T-1}$, we must replace (9) with the next two equations for the mean and variance:

$$E\left(\beta_{T-1} \,|\, \tilde{\beta}_T, \Re_{T-1}\right) = E\beta_{T-1|T-1} + \frac{\varphi V\beta_{T-1|T-1}}{\sigma_\eta^2 + \varphi^2 V\beta_{T-1|T-1}} \{\tilde{\beta}_T - [(1-\varphi)\beta + \varphi E\beta_{T-1|T-1}]\} \qquad \text{(11A)}$$

and

$$V\left(\beta_{T-1} \,|\, \tilde{\beta}_T, \Re_{T-1}\right) = V\beta_{T-1|T-1}\left(1 - \frac{\varphi^2 V\beta_{T-1|T-1}}{\sigma_\eta^2 + \varphi^2 V\beta_{T-1|T-1}}\right) \qquad \text{(11B)}$$

Given these moments we may generate $\tilde{\beta}_{T-1}$ from a normal distribution with mean and variance as described in Equations (11), and then work sequentially backwards to obtain the time sequence of betas: $(\tilde{\beta}_{T-2}, \ldots, \tilde{\beta}_1)$. Conditional on this sample, the unknown parameters $\Theta = \left((\alpha,\, \sigma_\varepsilon^2);\, (\phi, \beta, \sigma_\eta^2)\right)$ have easily derived posterior distributions. To show these results, define the excess return variable: $y_t = (R_t - R_{f,t-1}) - \beta_t(R_{m,t} - R_{f,t-1})$ for t=1, … , T. Then, assuming vague priors, the full conditional posterior distribution for $(\alpha, \sigma_{\varepsilon}^{2})$ are the same as (4A) and (4B). The latent variables $(\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_T)$ are sufficient statistics for the AR(1) model parameters. Thus, given the pseudo sample, inferences about $(\phi, \beta, \sigma_{\eta}^{2})$ may be obtained from the Markov chain independently of the data.

We need the deviations from the long run mean beta: $y_t = \tilde{\beta}_t - \beta$ for t=1, … , T. Then, the likelihood function for this sample -- expressed as a function of $(\phi, \sigma_{\eta}^{2})$ is proportional to:

$$L(\varphi, \sigma_{\eta}^{2}) \propto (\sigma_{\eta}^{2})^{-(T-1)/2} \exp\left[-\frac{1}{2\sigma_{\eta}^{2}} \sum_{t=2}^{T} \left(y_t - \varphi\, y_{t-1} \right)^2 \right] \tag{12}$$

Bayes’ theorem leads to a truncated normal posterior density for the autoregressive parameter with mean and variance:

$$E\varphi = V\varphi \left[\frac{1}{10^2} + \frac{1}{\sigma_{\eta}^{2}} \sum_{t=2}^{T} y_t y_{t-1} \right] \text{ and } V\varphi = \left(\frac{1}{10^2} + \frac{1}{\sigma_{\eta}^{2}} \sum_{t=2}^{T} y_t^2 \right)^{-1} \tag{13}$$

For the innovation variance, the full conditional posterior distribution of σ_{η}^{2} may be shown to be: $IG(C_1 + \frac{T-1}{2}, C_2 + \frac{1}{2}\sum_{t=2}^{T}(y_t - \phi\, y_{t-1})^2)$.

Last, for the long run mean of beta, define the filtered observations as: $y_t = \tilde{\beta}_t - \phi \tilde{\beta}_{t-1}$ for t=2, … , T. It is clear from the AR(1) model that y_t is an *iid* sequence of normal random variables with mean $\beta(1-\phi)$ and variance σ_η^2. Thus, the full conditional posterior distribution is normal with mean and variance:

$$E\beta = V\beta \left[\frac{1}{\sigma_\eta^2} \sum_{t=2}^{T} (1-\varphi) y_t \right] \text{ and } V\beta = \left(\frac{(T-1)(1-\varphi)^2}{\sigma_\eta^2} + \frac{1}{10^2} \right)^{-1} \quad (14)$$

Given new values of the model parameters and the sequence of market factor loadings, a Monte Carlo chain for the cost of capital may be generated. The MCMC algorithm consists of the following steps:

Algorithm 3: CAPM CoEC based on a Mean-Reverting (MR) market beta.

1. Generate a "pseudo" time series beta: $\tilde{B} = (\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_T)$.

2. Using the long run average of the risk free rate $\bar{R}_f$ *= 0.0363 and market risk premium (RP)* $\bar{R}_m - \bar{R}_f$ *= 0.0794, generate period t Bayesian CoEC as:* $C_t^{MR\ CAPM} = \bar{R}_f + (\bar{R}_m - \bar{R}_f)\beta_t$.

3. Go back to step 1 and repeat K times.

For the initial iteration (k=0) we set the three unknown parameters Θ equal to the mean and variance of 36-month rolling OLS estimates. For each firm, we generate K=2,000 Bayes estimates of the CoEC. To allow proper mixing we discard the first 1,000 estimates; also, we keep every fifth observation and obtain 200 final CoEC estimates.

2.3.2 MCMC for three-factor cost of equity capital

The benchmark model for the Fama and French (1993) three-factor CoEC assumes constant loadings as follows. Fama and French (1993) propose an empirical three-factor asset pricing model:

$$R_t - R_{f,t-1} = \alpha + \beta_m (R_{m,t} - R_{f,t-1}) + \beta_{smb} R_{smb,t} + \beta_{hml} R_{hml,t} + \varepsilon_t \tag{15}$$

The idiosyncratic returns ε_t are once again modeled as a sequence of identically, independently distributed normal variables with mean zero and variance σ_ε^2. For each firm, the three factor model contains five unknown parameters: $\Theta = \left((\alpha, \sigma_\varepsilon^2), (\beta_m, \beta_{smb}, \beta_{hml})\right)$.

For the first block $(\alpha, \sigma_\varepsilon^2)$, the prior distribution is the same as for the CAPM. For the vector of factor loadings $\mathrm{B} = (\beta_m, \beta_{smb}, \beta_{hml})$ we assume a multivariate normal density again with 0 mean and variance matrix $10^2 I_3$, where I_3 is the 3x3 identity matrix. Thus, the prior joint distribution for the model parameters is given by the product: $\pi(\Theta) = N\left(0, 10^2\right) \times IG(0.01, 0.01) \times N\left(0, 10^2 I_3\right)$. To obtain the likelihood function for $(\alpha, \sigma_\varepsilon^2)$, conditional on the three factor loadings, we define unexpected return variable: $y_t = R_t - R_{f,t-1} - [\beta_m (R_{m,t} - R_{f,t-1}) + \beta_{smb} R_{smb,t} + \beta_{hml} R_{hml,t}]$ for t=1, … , T. It is clear from the three factor model assumptions that y_t is an *iid* sequence of normal random variables with mean α , and variance σ_{ε}^{2}. Thus, Equations (4A) and (4B) provide, respectively, the full conditional posterior distribution for these two parameters.

The second parameter block is for the factor loading (column) vector $\mathrm{B} = (\beta_m, \beta_{smb}, \beta_{hml})'$. Define the excess return variables: $y_t = (R_t - R_{f,t-1}) - \alpha$, and the Tx3 matrix Z with t[th] row given by the factor returns $z_t = (R_{m,t} - R_{f,t-1}, R_{smb,t}, R_{hml,t})$ for t=1, … , T. Then, it is clear from the three-factor model assumptions that the conditional likelihood is given by:

$$L(B) \propto (\sigma_{\varepsilon}^{2})^{-T/2} \exp\left[-\frac{1}{2\sigma_{\varepsilon}^{2}} (Y - Z\,\mathrm{B})'(Y - Z\,\mathrm{B}) \right] \tag{16}$$

Multiplying the prior by this likelihood, we obtain for the three factor loadings a multivariate normal full conditional posterior distribution with mean and variance matrix:

$$EB = VB \times \left(\frac{1}{\sigma_{\varepsilon}^{2}} Z'Y \right) \text{ and } VB = \left(\frac{1}{10^{2}} I_3 + \frac{1}{\sigma_{\varepsilon}^{2}} Z'Z \right)^{-1} \tag{17}$$

To obtain the cost of capital, the Gibbs sampling algorithm consists of the following steps:

Algorithm 4: CoEC based on three-factors (3F) with constant factor loadings

1. Draw B = ($\beta_m, \beta_{smb}, \beta_{hml}$) from a Normal distribution with mean and variance matrix given by Equations (17).

2. Use the long run average of the risk-free rate $\bar{R}_f$ *= 0.0363, market risk premium (RP)* $\bar{R}_m - \bar{R}_f$ *= 0.0794, SMB factor* $\bar{R}_{smb}$ *= 0.0366 and HML factor* $\bar{R}_{hml}$ *= 0.0473 to compute the CoEC based on the three factor model for each firm:* $C_{3F} = R_f + (R_m - R_f)\beta_m + R_{smb}\beta_{smb} + R_{hml}\beta_{hml}$.

3. Go back to step 1 and repeat K times.

To start sampling, we set all unknown parameters in Θ equal to the full sample OLS estimates for each firm. As before, we generate K=2,000 CoEC estimates, discard the first 1,000 estimates and keep every fifth estimates thereafter. The remaining 200 observations represent the final sample from the posterior distribution of the CoEC implied by the three-factor model with constant factor loadings.

Contrary to the implicit assumption of time-invariant loadings in (10), Arifin (2013) suggests that loadings may vary with time. Of the three factors, SMB and HML loadings appear to vary less than the loading of market risk premium. In this study, we explicitly model market risk premium loading as a RW process while assuming that SMB and HML loadings to be time-invariant. The Fama-French (1993) three-factor asset pricing model with a time varying market risk premium is given by

$$R_t - R_{f,t-1} = \alpha + \beta_{m,t}(R_{m,t} - R_{f,t-1}) + \beta_{smb}R_{smb,t} + \beta_{hml}R_{hml,t} + \varepsilon_t \qquad (18A)$$

and

$$\beta_{m,t} = \beta_{m,t-1} + \eta_t \tag{18B}$$

for t = 1, … T, where $\eta_t \sim N\left(0, \sigma_\eta^2\right)$. In addition, we assume that η_t is uncorrelated with the idiosyncratic returns ε_t . For each firm, the three-factor model contains five unknown parameters: $\Theta = \left(\alpha, \sigma_\varepsilon^2,\ \beta_{smb}, \beta_{hml}, \sigma_\eta^2\right)$. The prior joint distribution for the model parameters is given by the product: $\pi(\Theta) = N\left(0,\ 10^2\right) \times IG\left(C_1, C_2\right) \times N\left(0,\ 10^2 I_2\right) \times IG\left(C_1, C_2\right)$.

To estimate this model, we need to generate the market factor loadings $(\tilde{\beta}_{m,1}, \dots, \tilde{\beta}_{m,T})$. But it turns out that the required steps are exactly the same as in the MCMC for CAPM CoEC with RW loading provided we define the excess return variable as: $y_{t+1} = (R_{t+1} - R_{f,t}) - [\alpha + \beta_{smb} R_{smb,t+1} + \beta_{hml} R_{hml,t+1}]$. After this step is completed, to obtain the posterior distribution for the first block of parameters – $(\alpha, \sigma_\varepsilon^2)$, set: $y_t = R_t - R_{f,t-1} - [\beta_{m,t}(R_{m,t} - R_{f,t-1}) + \beta_{smb} R_{smb,t} + \beta_{hml} R_{hml,t}]$ for t = 1, … , T, and generate random draws from Equations (4A) and (4B).

The second parameter block is for the factor loading (column) vector $\mathrm{B}_j = (\beta_{j,smb}, \beta_{j,hml})'$. Set the variables: $y_t = (R_t - R_{f,t-1}) - [\alpha + \beta_{m,t}(R_{m,t} - R_{f,t-1})]$, and the Tx2 matrix Z with t^{th} row defined as $Z_{j,t} = (R_{smb,t}, R_{hml,t})$ for t=1, … , T. Then, it is clear from the model assumptions that the loading vector B is bivariate normal with mean and variance as:

$$E\,B = V\,B \times \left(\frac{1}{\sigma_\varepsilon^2} Z'Y \right) \text{ and } V\,B = \left(\frac{1}{10^2} I_2 + \frac{1}{\sigma_\varepsilon^2} Z'Z \right)^{-1} \tag{19}$$

Last, for the variance of the beta innovations (σ_η^2), define the filtered observations $y_t = \tilde{\beta}_t - \tilde{\beta}_{t-1}$ for t=2, … , T. Then, y_t is an *iid* sequence of normal random variables with 0 mean and variance σ_η^2. Thus, the full conditional posterior distribution is given by: $IG(C_1 + \frac{T-1}{2},\, C_2 + \frac{1}{2}\sum_{t=2}^{T} y_t^2)$.

Given new values of the three-factor model parameters: $\Theta = \left(\alpha, \sigma_\varepsilon^2,\, \beta_{smb}, \beta_{hml}, \sigma_\eta^2\right)$, and the sequence of market factor loadings $(\tilde{\beta}_{m,1}, \ldots, \tilde{\beta}_{m,T})$, a chain for the three-factor cost of capital may be generated easily. The MCMC algorithm consists of the following steps:

Algorithm 5: CoEC based on the three-factor model with Random Walk beta.

1. Draw B = $(\beta_{smb}, \beta_{hml})$ from a Normal distribution with mean and variance matrix given by Equations (19). Generate a "pseudo" time series beta: $\tilde{B}_m = (\tilde{\beta}_{m,1}, \tilde{\beta}_{m,2}, \ldots, \tilde{\beta}_{m,T})$.

2. Use the long run average of the risk-free rate $\bar{R}_f$ = 0.0363, market risk premium (RP) $\bar{R}_m - \bar{R}_f$ = 0.0794, SMB factor $\bar{R}_{smb}$ = 0.0366 and HML factor $\bar{R}_{hml}$ = 0.0473 to compute Bayesian CoEC for period t: $C_t^{3F} = R_f + (R_m - R_f)\beta_{m,t} + R_{smb}\beta_{smb} + R_{hml}\beta_{hml}$

3. Go back to step 1 and repeat K times.

Initial values are obtained from 36-month rolling OLS estimates. In obtaining of our initial values, we use standard rolling OLS procedure that implicitly assumes that SMB and HML loading changes over time – differing from our MCMC model assumption. We assume that the effect of the initial values will dissipate in the burn-in period. The initial values for our MCMC simulation are set as the mean and variance of the rolling OLS estimates. We then generate K=2,000 estimates and keep every fifth observation after 1,000 burn-in draws. The resulting 200 estimates is our final sample to obtain CoEC and parameters estimates.

The analog of the Fama-French three-factor asset pricing model with a time varying AR(1) market beta is

$$R_t - R_{f,t-1} = \alpha + \beta_{m,t}(R_{m,t} - R_{f,t-1}) + \beta_{smb} R_{smb,t} + \beta_{hml} R_{hml,t} + \varepsilon_t \qquad \text{(20A)}$$

and

$$\beta_{m,t} = (1-\varphi)\beta_m + \varphi \beta_{m,t-1} + \eta_t \qquad \text{(20B)}$$

For each firm, this model contains seven parameters: $\Theta = \left((\alpha, \sigma_\varepsilon^2),\ (\beta_{smb}, \beta_{hml}), (\phi, \beta_m, \sigma_\eta^2)\right)$. The joint prior distribution is given by the product: $\pi(\Theta) = N\left(0,\ 10^2\right) \times IG(C_1, C_2) \times N\left(0,\ 10^2 I_2\right) \times TN\left(0, 10^2\right) \times N\left(0, 10^2\right) \times IG(C_1, C_2)$.

To generate the market factor loadings $(\tilde{\beta}_{m,1},\dots,\tilde{\beta}_{m,T})$, we set the excess return variable as: $y_{t+1}=(R_{t+1}-R_{f,t})-[\alpha+\beta_{smb}R_{smb,t+1}+\beta_{hml}R_{hml,t+1}]$, and follow the same steps as in MCMC for CAPM CoEC with AR(1) market beta. Similarly, to obtain autoregressive parameters φ, β_m and σ_η^2, we also follow the same steps as in MCMC for CAPM CoEC with AR(1) market beta. After this step is completed, to obtain the posterior distribution for the first block of parameters -- $(\alpha,\sigma_\varepsilon^2)$, set: $y_t=R_t-R_{f,t-1}-[\beta_{m,t}(R_{m,t}-R_{f,t-1})+\beta_{smb}R_{smb,t}+\beta_{hml}R_{hml,t}]$ for t=1, … , T, and generate random draws from Equations (4A) and (4B).

The second parameter block is for the factor loading (column) vector $\mathrm{B}_j=(\beta_{j,smb},\beta_{j,hml})'$. Set the variables: $y_t=(R_t-R_{f,t-1})-[\alpha+\beta_{m,t}(R_{m,t}-R_{f,t-1})]$, and the Tx2 matrix Z with t[th] row defined as $Z_{j,t}=(R_{smb,t},R_{hml,t})$ for t=1, … , T. Then, Equation (19) describes the loading vector B.

Last, for the variance of the beta innovations (σ_η^2), define the filtered observations $y_t=\tilde{\beta}_t-\tilde{\beta}_{t-1}$ for t=2, … , T. Then, y_t is an *iid* sequence of normal random variables with zero mean and variance σ_η^2. Thus, the full conditional posterior distribution is given by: $IG(C_1+\frac{T-1}{2}, C_2+\frac{1}{2}\sum_{t=2}^{T}y_t^2)$.

Given new values of the model parameters and the sequence of market factor loadings $(\tilde{\beta}_{m,1}, \ldots, \tilde{\beta}_{m,T})$, a chain for the three-factor cost of capital may be generated easily. The MCMC algorithm consists of the following steps:

Algorithm 6: COC based on the three-factor model with mean reverting beta.

1. Draw B = $(\beta_{smb}, \beta_{hml})$ *from a Normal distribution with mean and variance matrix given by Equations (19). Generate a "pseudo" time series beta:* $\tilde{B}_m = (\tilde{\beta}_{m,1}, \tilde{\beta}_{m,2}, \ldots, \tilde{\beta}_{m,T})$.

2. Use the long run average of the risk-free rate $\bar{R}_f$ *= 0.0363, market risk premium (RP)* $\bar{R}_m - \bar{R}_f$ *= 0.0794, SMB factor* $\bar{R}_{smb}$ *= 0.0366 and HML factor* $\bar{R}_{hml}$ *= 0.0473 to compute Bayesian CoEC for period t:* $C_t^{3F} = R_f + (R_m - R_f)\beta_{m,t} + R_{smb}\beta_{smb} + R_{hml}\beta_{hml}$

3. Go back to step 1 and repeat K times.

As before, we obtained initial values using 36-month rolling OLS estimates. We obtained 2,000 simulated CoEC estimates, throw away the first 1,000 estimates and keep every fifth observations afterwards. We then obtained 200 final CoEC estimates.

2.4 Forecasting Power of the Cost of Equity Capital

We are interested in out-of-sample performance of our dynamic models. CoEC forecast is important in practice. For example, valuation requires forecast of discount rates. From the perspective of a manager valuing short-term cash flows, a recent CoEC is a better discount rate to use. For valuation of distant cash flows however, CoEC dynamics over time maybe important.

If CoEC is mean reverting, we expect AR(1) models to obtain better forecasting fit. Alternatively, RW models may provide a better out-of-sample fit. In what follows, we investigate 1-month, 1-year and 5-year forecast horizon. We start by exploring out-of-sample fit of single-factor CoEC, followed by forecasting fit of three-factor CoEC.

2.4.1 Forecast based on the single-factor cost of equity capital

We investigate out-of-sample fit of single-factor CoEC as follows. For each firm i, we obtain a monthly time series of 1-month forecast error as

$$\varepsilon(1)_{t+1,i} = R_{t+1,i} - \hat{R}_{t+1,i} = R_{t+1,i} - \psi_{t+1,i}(\bar{R}_m - \bar{R}_f) \tag{21}$$

where $\psi_{t+1,i}$ is the expected market risk premium loading for firm i at time t+1 obtained from our MCMC models with time varying risk loadings. In MCMC for CAPM with constant loading model, $\psi_{t+1,i} = b_i$, in other words, market risk premium loading is time invariant. On the other hand, $\psi_{t+1,i} = b_{t,i}$ when beta is a RW process and $\psi_{t+1,i} = (1-\varphi)\bar{b} + \varphi b_{t,i}$ when beta is an AR(1) process. Additionally, $R_{t+1,i}$ is the annual realized return for firm i at time t+1, $\bar{R}_m$ is the long run average of the annualized CRSP value-weighted return and $\bar{R}_f$ is the long run average of the risk free rate. We set t to t+1 as one month and annualized by multiplying with 12.

An issue with our forecasting test is that we only have a limited data. We therefore use the whole sample to estimate $\psi_{t+1,i}$. For example, assuming a RW beta process, we first fit the MCMC model using the whole sample to obtain a time series of RW beta. We then use the resulting RW beta time series to forecast discount rate. Similarly, assuming an AR(1) beta process, we fit the MCMC model using the whole sample. We then use the resulting AR(1) beta time series to forecast discount rate. We follow identical approach assuming a constant beta. A limitation of our approach is a partiality toward a better forecasting fit.

From the time series of 1-month forecast error $\varepsilon(1)$, we obtain τ-year absolute forecast error as

$$\varepsilon\, abs(\tau)_{t+1,i} = \left| \sum_{j=1}^{\tau} \varepsilon(1)_{t+1-j,\, i} \right| \tag{22}$$

where τ = 12 for 1-year forecast horizon and τ = 60 for the 5-year forecast horizon. We then calculate the time series mean and standard deviation of the 1-month, 1-year and 5-year look ahead absolute forecast errors.

Table 3 presents the mean absolute forecast error of our 60-firm sample for MCMC for CAPM with constant loading (vague prior), MCMC for CAPM with RW market risk premium loading and MCMC for CAPM with AR(1) market risk premium loading. The null model is MCMC for CAPM with constant loading. In other words, we are interested in a lower mean absolute forecast error under dynamic beta specifications. On average, explicit modeling of beta over time does not appear to affect out-of-sample accuracy. The mean absolute forecast error for 1-month forecast is around 0.6 for all MCMC models. For 1-year forecast horizon, the mean forecast error is 2.3 for all models. The mean forecast error is 6.7 for all models when we forecast 5 years ahead.

At the firm-level, modeling beta dynamic over time appears to improve distant forecast accuracy for about half of our sample – the impact varies from firm to firm however. Forecasting 1 month ahead, Table 3 shows very little improvements in forecast accuracy at the firm level under both RW and AR(1) beta process – any forecast improvements are under 1%. At 1-year forecast horizon, 33 firms out of 60 reports a lower mean absolute forecast error when we model beta as RW – mean absolute forecast error decreases anywhere from 7.12% for firm SKT to 0.07% for firm ESS. Modeling beta as an AR(1) process with 1-year forecast horizon, mean absolute forecast error decreases ranging from a 4.66% decrease for firm HCN to 0.02% decrease for firm HME against the null model – thirty-six firms improves their forecast accuracy. When we forecast 5 year ahead assuming a RW beta, 32 firms shows improved forecast accuracy – improvements range from 19.10% lower mean absolute forecast error for firm ARE to 2.17% lower mean absolute forecast error for firm ADC against the null model. Assuming an AR(1) beta, forecasting 5 year ahead obtains 36 firms with improved forecast accuracy – improvements range from 16.44% lower mean absolute forecast error for firm AIV to 0.6% for firm REG against the null of time invariant beta. Among firms that shows forecast improvements, Table 3 shows that the majority obtains a better forecast under the assumption of a RW beta. A total of 36 firms show 1-year forecast improvements when we explicitly model market risk loading dynamics – 21 firms show a better forecast under RW market risk loading.

The result of Table 3 also indicates that about half of our sample reports worse forecast accuracy when we assume a time varying beta. Given that we use the whole sample to infer a time series of beta due to limited data, a look-ahead bias should be partial toward a better out-of-sample accuracy – making the lack of out-of-sample accuracy more conspicuous. Our forecast result suggests that for approximately half of our sample, MCMC for CAPM with constant beta provide a satisfactory discount rate to value distant future cash flows.

Table 4 reports the standard deviation of absolute forecast error for our 60-firm sample. On average, 1-month and 1-year look ahead obtains similar forecast error dispersion – standard deviation of absolute forecast error is about 0.47 and 1.67 for forecast horizon of 1 month and 1 year respectively. Forecasting 5 year ahead however, both RW and AR(1) beta specification report larger forecast dispersion, albeit modestly, compared to the constant beta specification. Examination of firm-level results also indicate that dispersion vary from firm-to-firm, particularly for distant discount rate forecast. Assuming a RW beta and forecasting 5 year ahead, the effect on forecast error standard deviation varies from a 27.89% lower dispersion for firm AIV to a 44.05% higher dispersion for firm HCP. If we assume an AR(1) beta and forecast discount rate 5 year ahead, the standard deviation of forecast error varies from a 15.51% lower dispersion for firm PLD to a 28.38% higher dispersion for firm WRE. It does appear then, at least for some firms in our sample, a manager valuing distant cash flows has to balance between obtaining a lower forecast error average versus lower forecast error dispersion. For example, firm HCN reports a 16.75% and 12.28% lower average forecast error at the 5-year forecast horizon assuming a RW and AR(1) beta respectively. These lower average forecast error however are accompanied by a 38.86% and 28.03% increase in forecast error standard deviation when we look 5 years ahead.

2.4.2 Forecast based on the three-factor cost of equity capital

Analyzing forecasting power of three-factor CoEC, we start by obtaining a time series of 1-month forecast error as

$$\varepsilon(1)_{t+1,i} = R_{t+1,i} - \hat{R}_{t+1,i} = R_{t+1,i} - \psi_{t+1,i}(\bar{R}_m - \bar{R}_f\ ) - c_i(\overline{SMB}) - d_i(\overline{HML}\ ) \qquad (23)$$

Specification (23) above assumes that SMB and HML loadings are time invariant. As before, if we assume constant market risk premium loading, we then set $\psi_{t+1,i} = b_i$. Alternatively, if we assume a RW market risk loading, we set $\psi_{t+1,i} = b_{t,i}$ and if we assume an AR(1) market risk loading, we set $\psi_{t+1,i} = (1-\varphi)b + \varphi b_{t,i}$. Similar to the single-factor analysis, we use the whole sample to estimate $\psi_{t+1,i}$ due to limited data. In addition, $\overline{SMB}$ is the long run average of annual SMB factor and $\overline{HML}$ is the long average of annual HML factor. Long run average is obtained from 1927 to 2011. We set t to t+1 period as 1 month and annualized by multiplying with 12. After we obtained the time series of 1-month forecast error $\varepsilon(1)_{t+1,i}$, we use equation (22) to obtain τ-year absolute forecast error with τ = 12 for 1-year forecast horizon and τ = 60 for the 5-year forecast horizon. Finally, we obtain the time series mean and standard deviation of absolute forecast error.

Table 5 reports the mean absolute forecast error of our 60-firm sample. The base model is MCMC for Fama and French (1993) with constant market risk premium (vague prior). The alternative models are MCMC for Fama and French (1993) with RW and AR(1) market risk premium loading. Averaging over 60 firms, looking ahead 1 month forward appears to not be sensitive to market risk premium loading dynamic – the mean absolute forecast error is 0.63 for all MCMC models. Similarly for 1-year forecast horizon, time varying market risk premium loading does not appear to improve out-of-sample accuracy. At 5-year horizon however, modeling the dynamic of beta increases forecast error on average – mean absolute forecast error forecast error averaging over our 60-firm sample increases by 6.05% when we model market risk premium loading as a RW process and increases by 17.86% assuming an AR(1) process.

Looking at firm-level results, modeling market risk premium loading as a RW process only have a limited effect at 1 month look ahead – twenty-seven firms show a lower mean absolute forecast error compared to the null CoEC model and all improvements are less than 1%. When we assume an AR(1) process however, only 5 firms report out-of-sample improvements and the improvements are modest. Extending the look ahead horizon to 1 year, if we assume a RW market risk premium loading, 23 firms show a lower mean absolute forecast error ranging from a 6.33% lower mean forecast error for firm BFS to a limited 0.01% lower mean forecast error for firm KIM. On the other hand, if we assume an AR(1) loading; only 5 firms report a lower mean absolute forecast error. Similarly, assuming a RW market risk premium loading looking ahead 5 years, Table 5 reports improvements on 23 out of 60 firms – ranging from 23.24% lower forecast error for firm OHI to 0.54% lower forecast error for firm TCO. In contrast, assuming an AR(1) process improves forecast error only for 9 firms – ranging from 38.75% lower forecast error average for firm FCH to 0.27% lower forecast error average for firm CBL. For Fama and French (1993) model therefore, within the subset of firms that shows forecast improvement, modeling market risk premium loading as RW appear to fit distant CoEC better. On the other hand, about half of our sample appears to have a time invariant market risk premium loading.

The standard deviation of absolute forecast error under MCMC for Fama and French (1993) models are presented in Table 6. Over our 60-firm sample, forecast error dispersion for 1 month and 1 year look ahead is similar. At longer horizon however, forecast error standard deviation increases when we model market risk premium loading as a time varying process. Firm-level results in Table 6 suggests that, at 5-year forecast horizon, the standard deviation of absolute forecast error ranges from a 18.77% lower dispersion for firm PPS to a 31.86% higher standard deviation for firm LHO when we assume a RW market risk premium loading. If we assume an AR(1) market risk premium loading, we obtain a 41.52% lower dispersion for firm DRE to a 94.04% higher dispersion for firm PLD. Given our earlier result preferring RW process to model market risk premium loading under the three-factor CoEC, an examination of Table 5 and 6 shows that firms obtaining an improvement in mean absolute forecast error do not obtain inferior forecast dispersion at the same time.

In sum, for single-factor CoEC, modeling time varying beta improves distant out-of-sample accuracy for about half of our sample based on firm-level results. Our analysis suggests that RW beta affords better distant forecast compared to AR(1) beta. At the same time, explicit modeling of beta dynamic increases distant forecast error dispersion at the firm-level. Care has to be taken therefore to balance obtaining a lower forecast error average with obtaining lower forecast error dispersion. In addition, we find that about half of our sample shows no improvements in forecast error – for these firms, our analysis points to time invariant beta.

Looking at firm-level three-factor CoEC forecast, modeling market risk loading time varying dynamic favors a RW process for distant forecasting and improves distant forecast for about half of our sample. Additionally, obtaining a lower mean absolute forecast error using RW market risk loading does not appear to increase forecast error standard deviation. Finally, the other half of our sample appears to obtain time invariant market risk premium loading even under three-factor CoEC.

2.5 Conclusion and Issues for Future Research

Cost of equity is an important concept in financial economics. A number of studies have looked at time variation in Equity REITs risk characteristic. In this study, we explicitly model time variation in market risk loading based on single-factor CAPM and three-factor Fama and French (1993) models under Bayesian specification for 60 Equity REITs. We additionally assume that SMB and HML loadings are time invariant. Empirically, we are interested in the out-of-sample fit of CoEC. Valuation requires an estimate of discount rate forecast. A manager interested in valuing near term cash flows may be interested in a more recent discount rate. Valuing distant cash flows however requires distant discount rate forecast. The accuracy of distant CoEC forecast may in turn depend on its dynamic over time.

Based on single-factor CAPM, we find an improved distant CoEC forecast for about half of our sample when we explicitly model market risk premium loading dynamic over time. Our evidence suggests that modeling market risk loading as a random walk is preferred over an autoregressive of order 1 process for the subset of firms with forecast improvements. We also find however that distant forecast dispersion increases when we model factor loading dynamic in the single-factor CoEC model. A managers deciding on distant discount rate for the purpose of valuation therefore have to balance between achieving a lower forecast error average and lower forecast error standard deviation. In addition, we also find that the other half of our sample obtains a smaller distant forecast error when we assume a time invariant market risk loading.

Evidence based on Fama and French (1993) three-factor model indicates that about half of our sample reports a smaller distant forecast error when we explicitly defined market risk loading dynamic. Again, random walk is preferred over an autoregressive process of order 1 within the subset of firms that exhibit forecast improvements. At the same time, we find that modeling market risk loading over time does not appear to increase distant forecast error standard deviation. Finally, as in the single-factor model, the other half of our sample reports a smaller distant forecast error under the assumption of time invariant market risk loading.

An interesting extension for future research is to enrich the dynamic of market risk loading. Our current evidence suggests that random walk is the preferred market risk loading dynamic. Technically however, a lack of mean reversion means that factor loading may grow or decline indefinitely. A casual observation of factor loading dynamic in the financial market suggest otherwise. A richer specification of loading dynamic over time may provide a solution. Another interesting extension is to specify the dynamic of SMB and HML loadings. A limitation of our current MCMC model is the assumption of time invariant SMB and HML loading. A time varying size and book-to-market factor loading may obtain additional improvements in CoEC estimates.

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Table 1 - Summary statistics of our 60-firm sample.

				Firm Returns				Volume	
Ticker	Firm Name	REIT Type	Mkt Val	Ave	Std	Skew	Kurt	Ave	Std
EPR	Entertainment Properties Trusts	Diversified	2,042	0.211	1.155	-0.110	8.648	1.559	3.609
WRE	Washington Real Estate Investment Trust	Diversified	1,807	0.112	0.781	-0.661	5.732	2.409	5.251
LXP	Lexington Realty Trust	Diversified	1,156	0.129	1.251	-0.029	15.270	3.687	8.439
CUZ	Cousins Properties Inc.	Diversified	665	0.041	1.106	-0.541	7.628	2.766	6.106
IRETS	Investors Real Estate Trusts	Diversified	610	0.077	0.545	0.006	3.594	0.922	2.136
HCP	HCP, Inc.	Health Care	16,928	0.188	0.965	-0.084	6.054	10.953	28.458
HCN	Health Care REIT, Inc.	Health Care	10,349	0.163	0.810	-0.095	3.411	5.588	13.883
OHI	Omega Healthcare Investors, Inc.	Health Care	1,996	0.148	1.666	0.180	5.576	3.058	6.756
HR	Healthcare Realty Trust	Health Care	1,447	0.113	0.936	-0.350	4.673	2.850	5.844
LTC	LTC Properties, Inc.	Health Care	936	0.168	1.067	-0.309	4.499	0.985	1.374
PLD	ProLogis Trust	Industrial	13,129	0.242	2.880	7.948	91.394	22.734	65.241
EGP	EastGroup Properties, Inc.	Industrial	1,177	0.154	0.806	-0.780	6.293	0.853	1.651
FR	First Industrial Realty Trust, Inc.	Industrial	886	0.102	1.512	-0.745	12.431	3.448	6.913
MNRTA	Monmouth Real Estate Investment Corp.	Industrial	359	0.139	0.624	0.704	6.698	0.262	0.549
HPT	Hospitality Properties Trust	Lodging	2,839	0.137	1.051	-0.687	9.247	4.999	11.261
LHO	LaSalle Hotel Properties	Lodging	2,028	0.224	1.687	1.898	21.447	2.852	6.839
FCH	FelCor Lodging Trust Incorporated	Lodging	380	0.075	2.117	0.724	7.139	4.262	8.044
EQR	Equity Residential Properties Trust	Multi-family	16,917	0.166	0.871	-0.566	4.690	13.660	28.908
AVB	AvalonBay Communities Inc.	Multi-family	12,418	0.184	0.869	-0.513	4.504	4.940	11.692
ESS	Essex Property Trust, Inc.	Multi-family	4,795	0.195	0.830	-0.195	3.581	1.880	4.251
CPT	Camden Property Trust	Multi-family	4,441	0.164	0.956	-0.525	5.365	3.456	7.333
BRE	BRE Properties, Inc.	Multi-family	3,801	0.144	0.897	-0.522	5.469	3.281	7.025
HME	Home Properties Inc.	Multi-family	2,779	0.153	0.809	-0.876	6.298	1.073	1.425
AIV	Apartment Investment & Management Co.	Multi-family	2,770	0.120	1.242	-0.966	9.268	7.875	17.651
MAA	Mid-America Apartment Communities, Inc.	Multi-family	2,366	0.173	0.786	-0.663	5.649	1.418	3.088
PPS	Post Properties, Inc.	Multi-family	2,268	0.111	1.000	-0.449	4.840	3.225	6.054
CLP	Colonial Properties Trust	Multi-family	1,823	0.160	1.436	2.270	28.998	2.695	5.764
AEC	Associated Estates Realty Corporation	Multi-family	675	0.173	1.171	-0.115	5.968	0.910	1.699

Table 1 - Continued

Ticker	Firm Name	REIT Type	Mkt Val	Firm Returns				Volume	
				Ave	Std	Skew	Kurt	Ave	Std
BXP	Boston Properties, Inc.	Office	14,726	0.179	0.906	0.183	8.334	6.819	15.618
SLG	SL Green Realty Corp.	Office	5,741	0.219	1.451	-0.004	10.921	4.958	13.780
ARE	Alexandria Real Estate Equities Inc.	Office	4,273	0.150	1.060	-0.178	12.882	2.132	5.512
DRE	Duke-Weeks Realty Corporation	Office	3,048	0.086	1.294	1.675	20.914	10.020	23.984
CLI	Mack-Cali Realty Corporation	Office	2,326	0.097	0.976	0.394	6.969	4.098	8.489
KRC	Kilroy Realty Corporation	Office	2,226	0.139	1.009	-0.553	5.204	2.436	4.959
HIW	Highwoods Properties, Inc.	Office	2,153	0.116	0.905	-0.629	4.208	3.868	7.145
OFC	Corporate Office Properties Trust	Office	1,530	0.170	0.859	-0.486	4.051	2.589	6.138
BDN	Brandywine Realty Trust	Office	1,288	0.131	1.737	3.453	33.582	5.623	13.508
PKY	Parkway Properties, Inc.	Office	217	0.037	1.204	-0.752	9.316	0.618	1.138
SPG	Simon Property Group, Inc.	Retail	37,888	0.212	1.010	0.165	10.412	11.967	30.576
MAC	Macerich Company, The	Retail	6,676	0.270	2.185	5.651	60.772	5.700	15.220
KIM	Kimco Realty Corporation	Retail	6,609	0.132	1.231	0.586	14.390	15.380	43.703
FRT	Federal Realty Investment Trust	Retail	5,762	0.185	0.801	-1.011	5.946	3.113	6.776
O	Realty Income Corporation	Retail	4,657	0.168	0.689	-0.016	3.620	4.004	10.135
TCO	Taubman Centers, Inc.	Retail	3,597	0.216	1.045	-0.348	7.230	3.750	7.321
REG	Regency Realty Corporation	Retail	3,383	0.142	1.001	-0.208	10.359	3.820	9.496
DDR	Developers Diversified Realty Corporation	Retail	3,371	0.158	1.649	0.785	20.190	10.638	27.935
WRI	Weingarten Realty Investors	Retail	2,637	0.128	1.222	1.209	16.292	4.552	10.771
SKT	Tanger Factory Outlet Centers, Inc.	Retail	2,542	0.222	0.791	0.643	8.192	1.972	4.537
CBL	CBL & Associates Properties, Inc.	Retail	2,329	0.290	2.757	6.818	71.222	5.781	14.883
EQY	Equity One, Inc.	Retail	1,947	0.148	0.799	-0.467	4.128	1.942	4.297
ALX	Alexander's Inc.	Retail	1,889	0.180	1.127	-0.356	5.302	0.164	0.195
GRT	Glimcher Realty Trust	Retail	989	0.177	1.609	0.262	11.188	2.551	5.517
AKR	Acadia Realty Trust	Retail	858	0.192	0.858	0.002	8.345	1.131	2.400
BFS	Saul Centers, Inc.	Retail	683	0.158	0.939	0.448	7.087	0.389	0.571
PEI	Pennsylvania Real Estate Investment Trust	Retail	581	0.172	1.874	2.537	24.900	2.222	5.071
RPT	Ramco-Gershenson Properties Trust	Retail	383	0.136	1.359	0.074	18.018	0.743	1.666

Table 1 – Continued

				Firm Returns				Volume	
Ticker	Firm Name	REIT Type	Mkt Val	Ave	Std	Skew	Kurt	Ave	Std
OLP	One Liberty Properties, Inc.	Retail	240	0.184	1.342	1.433	19.076	0.212	0.491
ADC	Agree Realty Corp	Retail	240	0.156	1.082	0.118	7.898	0.250	0.432
RPI	Roberts Realty Investors, Inc.	Retail	13	0.051	1.322	1.282	14.332	0.027	0.054
SSS	Sovran Self Storage, Inc.	Self Storage	1,189	0.145	0.900	-0.259	6.056	0.766	1.356

Notes: Table 1 reports summary statistics of the 60 firms in our sample. The period is January 1999 to December 2011. *REIT Type* denotes property focus. *Mkt Val* stands for market value. The market value is in millions of dollars as of 2011 Q4. We obtain market value as price per share multiplied by the number of shares outstanding obtained from CRSP. Firm returns signify annualized 1-month holding-period-return obtained from CRSP. We annualize returns by multiplying with 12. *Ave, Std, Skew* and *Kurt* signify average, standard deviation, skewness and kurtosis respectively. *Volume* is the monthly trading volume in millions of shares.

Table 2 – Summary statistics of Property Type portfolios, NAREIT index and factors.

Factors	Returns Ave	Std. Dev.	Std. Err.
R_f	0.036	0.031	0.003
R_m - R_f	0.079	0.208	0.023
SMB	0.037	0.142	0.015
HML	0.047	0.139	0.015

Notes: Table 2 reports summary statistics of annual factors from 1927 to 2011. We obtained factors from Dr. Kenneth French's website. *Ave*, *Std. Dev.*, *Skew* and *Kurt* denote average, standard deviation, skewness and kurtosis respectively. S*td. Err.* signifies standard deviation divided by square root of the number of data.

Table 3 – CAPM mean absolute forecast errors for our 60-firm sample.

	Model: Bayesian with vague prior			Model: Bayesian with random walk market risk premium loading			Model: Bayesian with AR(1) market risk premium loading		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Ticker	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
EPR	0.635	2.763	10.483	0.636	2.732	9.751	0.635	2.727	10.037
WRE	0.521	1.712	4.494	0.517	1.724	3.751	0.518	1.724	3.885
LXP	0.585	2.072	4.593	0.586	2.166	5.169	0.586	2.132	4.983
CUZ	0.647	1.934	5.562	0.649	2.003	6.271	0.648	1.971	5.925
IRETS	0.403	0.876	2.084	0.404	0.926	1.774	0.403	0.871	1.932
HCP	0.602	1.991	8.058	0.602	1.957	7.157	0.602	1.951	7.441
HCN	0.580	1.960	7.824	0.577	1.847	6.514	0.578	1.868	6.863
OHI	1.012	4.011	9.345	1.004	3.774	7.834	1.011	3.975	9.038
HR	0.629	1.965	4.269	0.631	2.044	4.711	0.630	1.986	4.363
LTC	0.719	3.024	10.943	0.713	2.882	10.031	0.718	3.001	10.692
PLD	0.642	2.190	6.033	0.642	2.252	7.076	0.642	2.224	6.840
EGP	0.545	1.692	6.739	0.542	1.645	6.041	0.544	1.661	6.456
FR	0.728	2.535	4.549	0.725	2.532	5.638	0.726	2.504	4.987
MNRTA	0.384	1.325	3.903	0.383	1.308	3.990	0.384	1.328	3.944
HPT	0.642	2.315	5.071	0.643	2.310	5.552	0.642	2.310	5.098
LHO	0.831	3.001	7.138	0.835	3.178	8.361	0.833	3.099	7.904
FCH	1.149	4.300	10.610	1.147	4.353	10.975	1.146	4.341	11.191
EQR	0.608	2.238	4.760	0.606	2.230	4.106	0.606	2.235	4.224
AVB	0.608	2.667	7.285	0.606	2.619	6.262	0.608	2.651	7.061
ESS	0.596	2.415	6.703	0.596	2.413	6.218	0.595	2.386	6.428
CPT	0.609	2.298	4.635	0.609	2.328	4.507	0.609	2.325	4.505
BRE	0.582	1.996	5.185	0.579	1.967	4.625	0.580	1.970	4.872
HME	0.549	1.639	5.938	0.549	1.641	5.009	0.549	1.639	5.715
AIV	0.716	2.341	2.071	0.716	2.276	1.680	0.716	2.313	1.730
MAA	0.532	1.972	6.719	0.529	1.933	5.945	0.531	1.950	6.483
PPS	0.635	2.671	2.372	0.631	2.616	2.437	0.633	2.621	2.303
CLP	0.522	2.282	4.385	0.523	2.291	4.400	0.523	2.276	4.353
AEC	0.731	2.669	7.565	0.731	2.665	7.729	0.731	2.692	7.890
BXP	0.581	2.409	7.631	0.581	2.355	7.120	0.580	2.384	7.309
SLG	0.722	3.394	7.851	0.724	3.490	7.600	0.724	3.471	7.556
ARE	0.542	1.762	6.790	0.540	1.719	5.493	0.541	1.703	6.125
DRE	0.640	1.936	5.509	0.643	2.037	6.509	0.640	1.926	5.638
CLI	0.617	1.658	5.027	0.620	1.767	5.829	0.617	1.660	5.095
KRC	0.705	2.655	5.887	0.704	2.632	6.056	0.704	2.616	5.798
HIW	0.663	1.640	3.518	0.661	1.607	3.079	0.661	1.606	3.145
OFC	0.609	2.646	9.898	0.607	2.693	9.313	0.608	2.639	9.830
BDN	0.714	2.109	6.537	0.717	2.282	7.955	0.716	2.203	7.390
PKY	0.696	1.872	5.354	0.696	1.998	6.199	0.696	1.922	5.712
SPG	0.625	2.519	9.687	0.622	2.449	8.585	0.622	2.456	8.772
MAC	0.709	2.814	8.193	0.708	2.788	8.321	0.707	2.768	8.136

Table 3 – Continued

Ticker	Estimation method: Bayesian with vague prior			Estimation method: Bayesian with random walk market risk premium loading			Estimation method: Bayesian with AR(1) market risk premium loading		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
KIM	0.609	2.477	7.022	0.609	2.512	7.586	0.609	2.500	7.444
FRT	0.548	2.291	11.400	0.549	2.257	10.481	0.548	2.273	10.792
O	0.491	1.892	6.188	0.489	1.933	5.347	0.489	1.921	5.497
TCO	0.623	2.727	10.945	0.619	2.661	10.419	0.621	2.683	10.600
REG	0.565	2.110	7.454	0.565	2.142	7.503	0.565	2.131	7.410
DDR	0.754	3.160	7.714	0.755	3.136	8.641	0.755	3.126	8.415
WRI	0.559	1.934	5.605	0.559	1.973	6.178	0.558	1.973	6.062
SKT	0.537	2.179	10.984	0.532	2.024	9.668	0.536	2.146	10.767
CBL	0.789	3.298	10.174	0.789	3.356	11.012	0.787	3.302	10.664
EQY	0.568	1.902	5.678	0.570	1.953	6.275	0.569	1.922	6.002
ALX	0.738	2.779	11.014	0.737	2.652	9.733	0.737	2.767	10.813
GRT	0.841	3.289	6.141	0.842	3.424	6.995	0.840	3.329	6.519
AKR	0.526	2.051	9.499	0.524	2.049	8.947	0.525	2.037	9.303
BFS	0.614	2.202	8.319	0.614	2.149	7.208	0.614	2.226	8.517
PEI	0.763	2.896	6.663	0.768	3.063	8.398	0.767	3.017	8.164
RPT	0.647	2.161	5.352	0.649	2.263	6.073	0.647	2.168	5.515
OLP	0.483	1.882	4.889	0.484	1.872	4.676	0.484	1.870	4.736
ADC	0.612	2.281	6.465	0.610	2.268	6.325	0.611	2.270	6.353
RPI	0.604	2.469	6.691	0.606	2.493	7.384	0.603	2.459	6.751
SSS	0.590	2.363	6.439	0.590	2.330	6.124	0.590	2.336	6.207
Ave	0.637	2.344	6.764	0.637	2.349	6.676	0.637	2.342	6.736

Notes: Table 3 reports the mean absolute forecast error of our 60-firm sample. The models presented are Markov Chain Monte Carlo (MCMC) for CAPM with vague prior (constant loading), MCMC for CAPM with random walk (RW) market risk premium loading and MCMC for CAPM with autoregressive order 1 (AR(1)) market risk premium loading. Under each MCMC model, we present the mean absolute forecast error with 1-month, 1-year and 5-year forecast horizon. *Ave* is average of 60 firms.

Table 4 – CAPM standard deviation absolute forecast errors for our 60-firm sample.

Ticker	Estimation method: Bayesian with vague prior			Estimation method: Bayesian with random walk market risk premium loading			Estimation method: Bayesian with AR(1) market risk premium loading		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
EPR	0.477	1.744	3.972	0.474	1.731	4.844	0.475	1.720	4.262
WRE	0.358	1.147	2.415	0.366	1.274	3.208	0.365	1.249	3.101
LXP	0.485	1.518	3.261	0.485	1.377	3.888	0.486	1.393	3.676
CUZ	0.482	1.367	2.334	0.484	1.447	2.849	0.485	1.419	2.718
IRETS	0.280	0.697	1.323	0.281	0.729	1.512	0.280	0.690	1.302
HCP	0.470	1.400	2.145	0.469	1.452	3.090	0.469	1.412	2.618
HCN	0.416	1.336	2.259	0.414	1.360	3.137	0.415	1.349	2.893
OHI	0.826	3.795	5.021	0.817	3.658	4.719	0.824	3.786	4.956
HR	0.486	1.362	3.586	0.488	1.446	3.731	0.486	1.390	3.676
LTC	0.537	2.560	5.951	0.536	2.515	6.371	0.537	2.555	5.957
PLD	0.453	1.850	3.654	0.456	1.884	3.103	0.456	1.878	3.088
EGP	0.369	1.059	2.979	0.369	0.976	3.419	0.368	1.026	3.088
FR	0.589	2.214	2.578	0.591	2.129	2.692	0.591	2.177	2.703
MNRTA	0.324	1.029	2.957	0.324	1.033	3.194	0.324	1.033	2.998
HPT	0.474	1.629	3.510	0.473	1.657	3.572	0.474	1.629	3.491
LHO	0.625	2.103	5.362	0.625	2.090	6.357	0.625	2.090	6.030
FCH	1.022	4.248	8.276	1.031	4.429	9.755	1.031	4.435	9.661
EQR	0.445	1.666	1.721	0.446	1.566	1.886	0.446	1.582	1.800
AVB	0.410	1.483	2.445	0.410	1.388	2.678	0.410	1.460	2.459
ESS	0.409	1.421	2.863	0.407	1.384	3.382	0.408	1.413	2.870
CPT	0.403	1.250	2.557	0.403	1.201	3.240	0.403	1.201	3.098
BRE	0.409	1.477	1.859	0.410	1.418	2.255	0.409	1.446	1.948
HME	0.381	1.230	1.437	0.380	1.116	1.874	0.381	1.184	1.488
AIV	0.508	1.731	1.715	0.505	1.659	1.236	0.507	1.694	1.458
MAA	0.394	1.398	2.753	0.395	1.317	3.022	0.394	1.384	2.766
PPS	0.450	2.070	1.528	0.449	1.938	1.778	0.449	1.992	1.692
CLP	0.365	1.706	2.426	0.361	1.616	3.071	0.361	1.624	2.937
AEC	0.513	1.935	2.355	0.511	1.900	2.359	0.513	1.956	2.356
BXP	0.380	1.466	3.498	0.379	1.459	3.813	0.380	1.446	3.529
SLG	0.545	2.234	4.455	0.544	2.144	5.065	0.545	2.151	4.938
ARE	0.372	1.088	3.353	0.373	1.014	3.881	0.371	1.048	3.398
DRE	0.468	1.644	2.531	0.469	1.706	2.901	0.469	1.687	2.834
CLI	0.378	1.200	2.210	0.379	1.270	2.502	0.379	1.210	2.120
KRC	0.476	1.651	4.152	0.478	1.677	3.987	0.477	1.654	3.931
HIW	0.491	1.096	1.620	0.494	1.091	1.671	0.494	1.083	1.576
OFC	0.474	1.551	6.006	0.478	1.509	6.282	0.474	1.547	5.977
BDN	0.545	2.061	3.444	0.549	2.168	3.884	0.548	2.133	3.773
PKY	0.590	1.581	2.985	0.597	1.613	3.245	0.593	1.616	3.298
SPG	0.412	1.438	3.851	0.410	1.356	4.780	0.411	1.367	4.608
MAC	0.506	1.802	6.097	0.505	1.819	5.876	0.506	1.814	5.926

Table 4 – Continued

	Estimation method: Bayesian with vague prior			Estimation method: Bayesian with random walk market risk premium loading			Estimation method: Bayesian with AR(1) market risk premium loading		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Ticker	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
KIM	0.483	1.669	3.454	0.485	1.655	3.098	0.484	1.661	3.110
FRT	0.374	1.438	3.951	0.370	1.359	4.710	0.371	1.374	4.520
O	0.350	1.221	3.019	0.354	1.212	3.833	0.353	1.204	3.707
TCO	0.433	1.583	3.128	0.433	1.488	3.512	0.433	1.556	3.177
REG	0.395	1.104	5.093	0.395	1.059	4.843	0.396	1.053	4.902
DDR	0.514	2.073	5.660	0.512	2.114	5.585	0.512	2.101	5.523
WRI	0.404	1.067	3.095	0.407	1.100	3.032	0.407	1.076	3.074
SKT	0.386	1.353	3.059	0.386	1.329	3.286	0.386	1.353	3.089
CBL	0.563	2.304	3.863	0.566	2.342	4.461	0.566	2.368	4.273
EQY	0.392	1.190	4.183	0.394	1.270	4.079	0.394	1.248	4.053
ALX	0.570	2.262	3.982	0.565	2.175	3.889	0.570	2.248	3.982
GRT	0.658	3.030	3.091	0.658	2.959	4.080	0.658	2.978	3.659
AKR	0.367	1.461	4.412	0.368	1.422	4.862	0.367	1.447	4.480
BFS	0.463	1.229	3.894	0.460	1.184	4.037	0.463	1.237	3.928
PEI	0.612	2.320	3.173	0.614	2.412	3.560	0.614	2.393	3.468
RPT	0.530	1.462	3.850	0.532	1.512	4.096	0.531	1.483	3.800
OLP	0.381	1.398	3.012	0.378	1.406	3.625	0.379	1.391	3.430
ADC	0.444	1.632	4.917	0.446	1.629	4.730	0.445	1.631	4.946
RPI	0.580	2.341	2.948	0.583	2.456	2.720	0.580	2.343	2.698
SSS	0.387	1.482	4.341	0.388	1.566	4.967	0.387	1.490	4.473
Ave	0.471	1.681	3.426	0.472	1.669	3.752	0.472	1.676	3.588

Notes: Table 4 reports the standard deviation of absolute forecast error of our 60-firm sample. The models presented are Markov Chain Monte Carlo (MCMC) for CAPM with vague prior (constant loading), MCMC for CAPM with random walk (RW) market risk premium loading and MCMC for CAPM with autoregressive order 1 (AR(1)) market risk premium loading. Under each MCMC model, we present the standard deviation of absolute forecast error with 1-month, 1-year and 5-year forecast horizon. *Ave* is average of 60 firms.

Table 5 – Fama and French (1993) mean absolute forecast errors for our 60-firm sample.

	Estimation method: Bayesian with market risk premium loading vague prior			Estimation method: Bayesian with random walk market risk premium loading			Estimation method: Bayesian with AR(1) market risk premium loading		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Ticker	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
EPR	0.629	2.548	7.720	0.631	2.576	7.872	0.634	2.703	9.765
WRE	0.512	1.502	2.337	0.512	1.573	2.826	0.516	1.594	3.271
LXP	0.582	1.930	3.895	0.583	1.973	4.506	0.584	2.004	4.151
CUZ	0.647	1.977	5.099	0.648	1.986	5.687	0.647	1.934	5.459
IRETS	0.402	0.848	1.316	0.402	0.871	1.712	0.403	0.858	1.706
HCP	0.600	1.763	6.050	0.600	1.823	6.080	0.602	1.993	8.074
HCN	0.579	1.727	5.778	0.576	1.723	5.382	0.580	1.913	7.447
OHI	1.008	3.888	6.859	1.005	3.813	5.265	1.012	4.011	9.344
HR	0.627	1.925	4.045	0.626	1.987	4.732	0.629	1.949	4.118
LTC	0.715	2.861	8.622	0.711	2.760	8.026	0.718	3.011	10.789
PLD	0.639	2.042	5.677	0.640	2.160	6.723	0.646	2.341	6.546
EGP	0.541	1.408	4.543	0.539	1.357	4.071	0.544	1.652	6.438
FR	0.728	2.454	4.688	0.725	2.372	5.067	0.728	2.515	4.478
MNRTA	0.382	1.228	3.216	0.381	1.204	3.314	0.384	1.300	3.707
HPT	0.638	2.207	4.937	0.638	2.204	5.121	0.643	2.346	5.163
LHO	0.825	2.753	6.028	0.832	2.976	7.490	0.833	3.094	7.870
FCH	1.162	4.566	16.594	1.162	4.615	16.304	1.148	4.289	10.164
EQR	0.603	2.057	2.512	0.602	2.058	2.263	0.608	2.243	4.812
AVB	0.602	2.487	4.565	0.602	2.465	3.821	0.609	2.671	7.334
ESS	0.589	2.173	4.163	0.590	2.163	3.997	0.595	2.376	6.331
CPT	0.603	2.107	2.852	0.604	2.172	3.453	0.609	2.306	4.719
BRE	0.581	1.792	2.541	0.579	1.746	2.427	0.581	1.941	4.551
HME	0.548	1.398	3.673	0.548	1.437	3.317	0.548	1.548	5.196
AIV	0.714	2.337	2.994	0.714	2.258	2.612	0.716	2.332	1.965
MAA	0.528	1.832	5.158	0.526	1.804	4.645	0.534	2.046	7.418
PPS	0.634	2.603	2.766	0.631	2.546	2.545	0.635	2.690	2.447
CLP	0.519	2.114	3.091	0.521	2.145	3.676	0.521	2.215	3.829
AEC	0.726	2.473	4.582	0.728	2.479	4.602	0.729	2.585	6.386
BXP	0.577	2.295	5.750	0.578	2.279	5.377	0.583	2.451	8.215
SLG	0.715	3.113	5.389	0.718	3.267	6.198	0.722	3.395	7.864
ARE	0.539	1.516	4.599	0.537	1.509	4.319	0.541	1.670	6.012
DRE	0.640	1.875	5.667	0.644	1.921	6.343	0.640	1.935	5.509
CLI	0.616	1.618	4.750	0.618	1.697	5.336	0.617	1.663	5.044
KRC	0.702	2.523	4.629	0.704	2.558	5.230	0.705	2.634	5.663
HIW	0.659	1.516	1.398	0.657	1.503	1.588	0.662	1.608	3.027
OFC	0.600	2.420	8.130	0.599	2.410	7.967	0.606	2.578	9.310
BDN	0.709	2.081	7.058	0.713	2.197	7.929	0.713	2.085	6.581
PKY	0.702	1.979	5.897	0.701	2.002	6.034	0.697	1.875	5.383
SPG	0.614	2.223	6.889	0.615	2.248	6.825	0.622	2.445	9.020
MAC	0.696	2.529	6.923	0.698	2.547	7.429	0.705	2.719	7.647

Table 5 – Continued

	Estimation method: Bayesian with market risk premium loading vague prior			Estimation method: Bayesian with random walk market risk premium loading			Estimation method: Bayesian with AR(1) market risk premium loading		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Ticker	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
KIM	0.605	2.285	6.414	0.605	2.284	6.893	0.609	2.483	7.040
FRT	0.541	2.005	8.921	0.543	2.008	8.348	0.546	2.200	10.636
O	0.484	1.686	4.125	0.484	1.775	4.161	0.488	1.800	5.270
TCO	0.614	2.451	8.556	0.613	2.433	8.510	0.624	2.781	11.380
REG	0.560	1.907	6.480	0.561	1.920	6.682	0.564	2.082	7.281
DDR	0.746	2.938	7.212	0.748	2.954	8.043	0.756	3.202	7.895
WRI	0.556	1.768	5.178	0.556	1.811	5.764	0.558	1.895	5.487
SKT	0.532	1.979	9.384	0.529	1.866	8.089	0.537	2.200	11.142
CBL	0.779	3.090	10.055	0.780	3.119	10.630	0.785	3.211	10.028
EQY	0.560	1.736	5.089	0.564	1.822	5.843	0.568	1.920	5.774
ALX	0.735	2.674	8.792	0.736	2.603	7.944	0.739	2.826	11.686
GRT	0.837	3.122	5.115	0.841	3.287	6.106	0.841	3.279	6.088
AKR	0.521	1.840	7.758	0.521	1.851	7.567	0.526	2.070	9.654
BFS	0.613	1.955	6.204	0.611	1.832	5.173	0.613	2.069	7.207
PEI	0.760	2.727	6.803	0.763	2.925	8.129	0.762	2.857	6.673
RPT	0.643	1.989	5.120	0.645	2.088	5.777	0.646	2.138	5.296
OLP	0.479	1.750	3.508	0.481	1.784	3.954	0.483	1.882	4.882
ADC	0.608	2.143	5.424	0.607	2.151	5.719	0.610	2.220	5.905
RPI	0.608	2.527	6.515	0.608	2.529	7.094	0.605	2.480	6.639
SSS	0.585	2.210	4.731	0.586	2.208	5.094	0.589	2.314	5.830
Ave	0.633	2.191	5.579	0.634	2.211	5.727	0.637	2.324	6.576

Notes: Table 5 reports the mean absolute forecast error of our 60-firm sample. The models presented are Markov Chain Monte Carlo (MCMC) for Fama and French (1993) three-factor model with market risk premium loading vague prior (constant market risk premium loading), MCMC for Fama and French (1993) three-factor model with random walk (RW) market risk premium loading and MCMC for Fama and French (1993) three-factor model with autoregressive order 1 (AR(1)) market risk premium loading. SMB and HML factors are assumed to be time invariant. Under each MCMC model, we present the mean absolute forecast error with 1-month, 1-year and 5-year forecast horizon. *Ave* is average of 60 firms.

Table 6 – Fama and French (1993) standard deviation of absolute forecast errors for our 60-firm sample.

	Estimation method: Bayesian with market risk premium loading vague prior			Estimation method: Bayesian with random walk market risk premium loading			Estimation method: Bayesian with AR(1) market risk premium loading		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
Ticker	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
EPR	0.473	1.520	3.916	0.471	1.557	4.491	0.475	1.682	3.972
WRE	0.363	1.060	1.741	0.367	1.141	2.077	0.360	1.095	2.144
LXP	0.485	1.401	2.504	0.483	1.303	2.767	0.485	1.456	2.801
CUZ	0.486	1.466	4.079	0.486	1.509	4.275	0.483	1.379	2.554
IRETS	0.281	0.637	1.002	0.282	0.663	1.016	0.280	0.671	1.202
HCP	0.465	1.267	2.126	0.466	1.345	2.775	0.470	1.401	2.145
HCN	0.410	1.271	2.259	0.412	1.272	2.835	0.414	1.324	2.259
OHI	0.831	3.886	4.410	0.824	3.822	3.853	0.826	3.795	5.020
HR	0.487	1.253	2.384	0.489	1.284	2.455	0.486	1.341	3.383
LTC	0.535	2.527	5.950	0.533	2.477	5.999	0.537	2.558	5.950
PLD	0.457	1.925	2.431	0.458	1.915	2.425	0.451	1.802	4.718
EGP	0.365	0.998	2.798	0.364	0.964	2.820	0.369	1.048	2.979
FR	0.590	2.302	4.524	0.589	2.234	4.351	0.589	2.222	2.814
MNRTA	0.322	0.953	2.361	0.322	0.957	2.616	0.323	1.010	2.833
HPT	0.476	1.632	2.577	0.477	1.605	2.793	0.474	1.632	3.771
LHO	0.625	2.130	3.387	0.624	2.176	4.466	0.625	2.113	5.716
FCH	1.034	4.775	8.453	1.038	4.824	9.065	1.021	4.204	8.179
EQR	0.445	1.596	1.190	0.446	1.574	1.258	0.445	1.667	1.726
AVB	0.409	1.335	2.445	0.407	1.312	2.469	0.410	1.486	2.446
ESS	0.408	1.363	2.759	0.406	1.341	2.843	0.408	1.411	2.863
CPT	0.404	1.239	1.933	0.404	1.225	2.286	0.403	1.252	2.575
BRE	0.402	1.387	1.686	0.402	1.355	1.614	0.407	1.452	1.859
HME	0.374	1.112	1.397	0.374	1.071	1.624	0.379	1.196	1.437
AIV	0.511	1.678	1.852	0.508	1.672	1.684	0.508	1.715	1.585
MAA	0.394	1.317	2.683	0.395	1.291	2.718	0.394	1.432	2.753
PPS	0.450	2.121	2.104	0.448	2.001	1.709	0.451	2.063	1.587
CLP	0.362	1.684	2.122	0.359	1.654	2.571	0.364	1.694	2.273
AEC	0.510	1.798	2.253	0.509	1.790	2.294	0.511	1.872	2.355
BXP	0.379	1.335	3.461	0.377	1.318	3.607	0.381	1.511	3.498
SLG	0.545	2.265	3.555	0.545	2.219	4.163	0.545	2.234	4.457
ARE	0.368	0.993	3.009	0.371	0.961	2.788	0.370	1.053	3.278
DRE	0.472	1.846	4.324	0.473	1.934	4.716	0.469	1.645	2.540
CLI	0.382	1.251	2.293	0.383	1.307	2.569	0.378	1.198	2.257
KRC	0.475	1.613	2.820	0.476	1.679	2.644	0.476	1.643	3.999
HIW	0.496	1.086	1.035	0.498	1.106	1.206	0.492	1.082	1.488
OFC	0.477	1.491	4.993	0.479	1.513	4.843	0.475	1.515	5.716
BDN	0.558	2.328	5.937	0.561	2.431	6.096	0.547	2.110	3.816
PKY	0.590	1.763	5.086	0.595	1.786	4.912	0.590	1.612	3.301
SPG	0.415	1.314	3.793	0.412	1.318	4.418	0.413	1.403	3.851
MAC	0.515	1.820	3.973	0.513	1.868	3.871	0.508	1.803	5.624

Table 6 – Continued

Ticker	Estimation method: Bayesian with market risk premium loading vague prior			Estimation method: Bayesian with random walk market risk premium loading			Estimation method: Bayesian with AR(1) market risk premium loading		
	Forecast horizon:			Forecast horizon:			Forecast horizon:		
	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr	1 mo	1 yr	5 yr
KIM	0.486	1.798	2.161	0.486	1.761	2.190	0.483	1.666	3.504
FRT	0.369	1.250	3.951	0.366	1.253	4.552	0.373	1.375	3.951
O	0.350	1.091	2.687	0.353	1.133	3.184	0.350	1.161	2.948
TCO	0.432	1.489	3.128	0.432	1.455	3.357	0.433	1.603	3.128
REG	0.396	1.086	3.771	0.395	1.069	3.504	0.395	1.097	4.959
DDR	0.521	2.155	3.769	0.517	2.137	4.003	0.513	2.062	5.899
WRI	0.406	1.169	2.142	0.408	1.176	2.387	0.405	1.079	2.787
SKT	0.383	1.239	3.059	0.384	1.226	3.152	0.386	1.365	3.059
CBL	0.578	2.597	5.042	0.578	2.600	5.200	0.568	2.401	4.131
EQY	0.398	1.192	2.740	0.397	1.257	3.029	0.392	1.193	4.292
ALX	0.567	2.095	3.982	0.563	2.102	3.871	0.572	2.307	3.982
GRT	0.658	3.019	4.011	0.658	3.023	4.327	0.658	3.030	3.079
AKR	0.364	1.381	4.409	0.364	1.373	4.583	0.367	1.469	4.412
BFS	0.456	1.176	3.506	0.454	1.148	3.155	0.459	1.195	3.706
PEI	0.616	2.491	4.155	0.618	2.488	3.710	0.613	2.343	3.071
RPT	0.532	1.524	2.473	0.534	1.558	2.734	0.530	1.470	3.719
OLP	0.380	1.361	2.564	0.378	1.367	3.011	0.381	1.397	3.011
ADC	0.444	1.542	3.499	0.446	1.576	3.504	0.444	1.588	4.465
RPI	0.578	2.341	4.114	0.582	2.459	3.416	0.580	2.339	3.207
SSS	0.387	1.380	3.801	0.388	1.488	4.129	0.387	1.440	4.234
Ave	0.472	1.669	3.209	0.472	1.674	3.350	0.471	1.673	3.421

Notes: Table 6 reports the standard deviation of absolute forecast error of our 60-firm sample. The models presented are Markov Chain Monte Carlo (MCMC) for Fama and French (1993) three-factor model with market risk premium loading vague prior (constant market risk premium loading), MCMC for Fama and French (1993) three-factor model with random walk (RW) market risk premium loading and MCMC for Fama and French (1993) three-factor model with autoregressive order 1 (AR(1)) market risk premium loading. SMB and HML factors are assumed to be time invariant. Under each MCMC model, we present the standard deviation of absolute forecast error with 1-month, 1-year and 5-year forecast horizon. *Ave* is average of 60 firms.

CHAPTER 3

On the Cost of Equity Capital of Real Estate Investment Trusts (REITs): Improving Cost of Equity Estimates

3.1 Introduction

Cost of equity capital (CoEC) is an important concept in financial economics, useful in capital budgeting decision and critical in valuation. Components of CoEC however have to be approximated, resulting to a possibly imprecise CoEC estimate. Using data on 48 industries, Fama and French (1997) shows that risk loadings and risk premiums are inaccurately estimated – leading to an imprecise CoEC estimate. Similarly, within a Bayesian setting, Pastor and Stambaugh (1999) suggest that uncertainty surrounding risk exposures and prices of risks is an important contributor to overall CoEC uncertainty. For Equity REITs, the inaccuracy is negligible nominally. Using 60 Equity REITs, we find an average single-factor Sharpe (1964) CAPM CoEC of 8.182% per year with an average uncertainty of 1.531% per year. Under Fama and French (1993) three-factor model, the average CoEC and its uncertainty using the same 60-firm data are 12.638% and 2.082% per year respectively. Nonetheless, CoEC is an important concept – warranting, we believe, an effort to obtain a more accurate estimate.

In this study, we approach CoEC uncertainty through a Bayesian framework. We note that Arifin (2013) finds considerable variations in CoEC estimates using our 60-firm sample. Given that our sample originates from the Equity REITs industry, it is reasonable to expect that the same systematic factors affect all firms. It is then possible to "borrow" information from other firms to obtain a "revised" risk loading for a particular firm and therefore an improved CoEC estimates for that firm. We do so formally through a Bayesian Hierarchical framework – effectively applying a Bayesian shrinkage approach. A number of studies have utilized additional information to improve risk estimates. An early study by Vasicek (1973) suggested the use of sample cross-sectional information to improve CAPM beta. Using 500 randomly selected New York Stock Exchange (NYSE) stock data, Young and Lenk (1998) show that utilizing Bayesian Hierarchical model improves estimation accuracy for model with exogenous market risk premium and size factors. A more recent paper by Cosemans, Frehen, Schotman and Bauer (2012) obtains a "hybrid" beta that combines rolling-window ordinary least squares beta with beta conditioned using firm-specific information. Tan (2012) finds that applying a Bayesian shrinkage approach obtains a more accurate out-of-sample industry cost of equity.

In this study, we investigate in-sample CoEC accuracy within the context of single-factor Sharpe (1964) CAPM and three-factor Fama and French (1993) model for Equity REITs. Our results are as follows. Using 60 Equity REITs as our sample, we find that using cross-sectional information to supplement a firm's information set only have a limited effect on Equity REITs CoEC. We compare Bayesian Hierarchical CoEC against two benchmark models: full sample ordinary least squares (OLS) CoEC and Bayesian with vague prior CoEC. In the single-factor CAPM model, the additional information has only a modest effect overall – estimated average CoEC is about 1.5% lower than the benchmark models with about 6% lower average CoEC uncertainty compared to the benchmark CoEC. Utilizing Bayesian Hierarchical model at the firm-level lowers CAPM CoEC uncertainty for the majority of our firms. The improvements in CoEC imprecision are however small. In addition, “borrowing” information does not obtain a more precise single-factor CoEC for all firms i.e. the effect of the additional information is inconsistent.

Our results using Fama and French (1993) three-factor model do not fare considerably better. The average three-factor Bayesian Hierarchical CoEC is 21% lower than the benchmark CoECs; average CoEC uncertainty is only about 2.5% lower than benchmark. Using a “revised” risk loading does lower CoEC imprecision for the majority of our 60-firm sample. The improvements are again small and inconsistent.

We discuss our data next. Our single-factor and three-factor Bayesian Hierarchical specifications, their respective CoEC estimates and discussions follow. We then conclude the paper.

3.2 Data

We obtain a list of Equity REITs from SNL and National Association of Real Estate Investment Trusts (NAREIT) as of December 2011. Imposing a requirement of 10-year continuous operating history from 1998 to 2011 results in a final list of 60 Equity REITs encompassing 8 different property types: Diversified, Health Care, Industrial, Lodging, Multi-family, Office, Retail and Self-storage. Data on these 60 firms have differing start date. For example, data on Boston Properties (BXP) starts July 1997 while data on Duke-Weeks Realty (DRE) starts on January

1993. We impose a common sample period from January 1999 to December 2011 to eliminate variations due to different sample period.

Table 1 presents the ticker, firm name, each firm's property type, market value, annualized firm returns (average, standard error, skewness and kurtosis) as well as monthly trading volume (average and standard deviation) for our 60-firm sample. Return data and trading volume were obtained from CRSP. We annualize by multiplying with 12. As shown in Table 1, our sample ranges from Roberts Realty Investors (RPI) with a market value of $13 million to Simon Property Group (SPG) with a market value of $38 billion. Annualized return averages 0.16 with average standard error of 0.09 over 60 firms. Standard error is calculated as the standard deviation divided by the square root of the number of data. Firm-by-firm standard error can be a bit larger – sample mean is further away from the population mean at the firm level. For example, the average and standard error of RPI's annual return is 0.051 and 0.106 respectively. In addition, our sample appears to be non-normal: 34 firms out of 60 exhibits negative skewness and all 60 firms exhibits kurtosis larger than 3. Firm returns are winsorized at 5% and 95% to minimize the effects of outliers. Furthermore, firms in our sample are actively traded. On average, 4 million shares are traded every month with a standard deviation of 9.68 million shares.

We use long run annual average from 1927 to 2011 as our factors. Data are obtained from Dr. Kenneth French's website. Risk premiums are difficult to measure; we therefore use long run data that spans several business cycles to obtain more accurate estimates. Fama and French (1997) use similarly long factor data from 1963 to 1994 in their study. Table 2 reports the summary statistics of factors used in our study. The average long run risk free rate in our study is 3.63% per year. The long run average of annual market risk premium $R_{m,t} - R_{f,t-1}$ factor is 7.94% with a standard deviation of 20.8% per year. The market risk premium factor is volatile from 1927 to 2011. The standard error for market risk premium is 2.3% per year – standard error is calculated as standard deviation of market risk premium divided by the square root of the number of time-series data. The additional factors needed for Fama and French (1993) three-factor CoEC are SMB (size) and HML (book-to-market) factor. SMB factor averages 3.66% per year from 1927 to 2011; the annual standard deviation is 14.2% while the standard error is 1.5% per year. The long run average, standard deviation and standard error of the book-to-market factor are 4.73% per year, 13.9% per year and 1.5% per year respectively. Similar to the market risk premium factor, the additional size and book-to-market factors vary considerably from 1927 to 2011.

3.3 Accuracy of Cost of Equity Estimates under Bayesian Hierarchical Framework

We consider cost of equity models under single-factor CAPM of Sharpe (1963) and three-factor model of Fama and French (1993). For each firm and model, we obtain two benchmark specifications. Our first benchmark CoEC is estimated via full sample OLS. Our second benchmark is estimated via Bayesian framework with vague prior. These two benchmark CoEC estimates are comparable. Our alternative CoEC is estimated via Bayesian Hierarchical framework. Arifin (2013) shows a wide variation in both the single-factor and three-factor CoEC estimates across our 60-firm sample. Given that our sample resides in the same industry, all 60 firms should be exposed to the same systematic factors. Within the Bayesian Hierarchical framework, we are able to formally "borrow" information from other firms, allowing us the possibility of improving CoEC estimate for each individual firm. We assume the additional information would lead to a refinement in the CoEC estimates; we therefore expect that Bayesian Hierarchical CoEC estimate to have a lower dispersion. In other words, Bayesian Hierarchical CoEC is expected to be more accurate. CoEC imprecision is defined following Fama and French (1997) specification which we will make clear in subsequent discussions.

At the risk of getting ahead of ourselves, our results however provide evidence against using cross-sectional information to improve Equity REITs CoEC. Using Bayesian Hierarchical framework does not categorically improve the accuracy of CoEC. The 60-firm average annual CAPM CoEC is about 1.5% lower than the benchmark model; CoEC dispersion is about 6% lower than those of the benchmarks. At the firm level, annual Bayesian Hierarchical CoEC is more precise for the majority of our sample – these improvements are however small. Moreover, some firms reports a higher imprecision when we use the Bayesian Hierarchical framework. Likewise, "borrowing" information in three-factor Bayesian Hierarchical CoEC framework lowers the average annual discount rate by 21% compared to the benchmarks; CoEC imprecision improves by about 2.5%. Looking at the results firm-by-firm, we find that while CoEC uncertainty per year decreases for the majority of our sample using Bayesian Hierarchical specification, the improvements are small and inconsistent. We proceed as follows. Single-factor CoEC is analyzed next, followed by analysis of Fama and French (1993) three-factor CoEC.

Given excess return on firm j and the market portfolio, we have

$$R_{j,t} - R_{f,t-1} = \alpha_j + \beta_{j,m}\left(R_{m,t} - R_{f,t-1}\right) + \varepsilon_{j,t} \tag{1}$$

where $\varepsilon_{j,t} \sim N\left(0, \sigma^2_{j,\varepsilon}\right)$. Within the Bayesian Hierarchical framework, the single factor model for each firm contains three unknown parameters: $\Theta_j = \left(\alpha_j, \sigma^2_{j,\varepsilon}, \beta_{j,m}\right)$. Let $\mathrm{B}_j = \left(\beta_{j,m}\right)$, the firm *j* parameters may be summarized by the vector: $\Theta_j = \left(\alpha_j, \sigma^2_{j,\varepsilon}, B_j\right)$. The hyper parameters for our models are $\alpha_j \sim N\left(m_\alpha, \upsilon^2_\alpha\right)$, $B_j \sim N\left(m_B, \Omega_B\right)$ and $\sigma^2_{j,\varepsilon} \sim IG\left(C_1, C_2\right)$.

For each firm *j*, we obtain posterior distribution of α_j as follows. To obtain the likelihood function, conditional on beta, define the unexpected return variable: $y_{j,t} = (R_{j,t} - R_{f,t-1}) - \beta_{j,m}(R_{m,t} - R_{f,t-1})$ for time periods t=1, … , T. It is clear from assumptions in equation (1) that $y_{j,t}$ is an *iid* sequence of normal random variables with mean α_j, and variance $\sigma^2_{j,\varepsilon}$. Therefore, the likelihood function for the sample $Y_j = \left(y_{j,1}, y_{j,2}, \ldots, y_{j,T}\right)$, expressed as a function of $(\alpha_{j,}, \sigma^2_{j,\varepsilon})$ only, is given by:

$$L\left(\alpha_j, \sigma^2_{j,\varepsilon}\right) = \left(2\pi\, \sigma^2_{j,\varepsilon}\right)^{-T/2} \exp\left[-\frac{1}{2\sigma^2_{j,\varepsilon}} \sum_{t=1}^{T} \left(y_{j,t} - \alpha_j\right)^2\right] \tag{2}$$

If we use Bayes' theorem, then we obtain a normal full conditional posterior distribution for the pricing error α (given the data and the remaining parameters) with mean and variance:

$$E\alpha_j = W_j \bar{y}_j + \left(1 - W_j\right) m_\alpha \quad \text{and} \quad V\alpha_j = \left(1 - W_j\right) \upsilon^2_\alpha \tag{3}$$

where $W_j = \dfrac{\upsilon_\alpha^2 T}{\upsilon_\alpha^2 T + \sigma_{j,\varepsilon}^2}$ and $\bar{y}_j$ is the time series sample mean of $y_{j,t}$.

The second parameter block is for the factor loading (column) vector $\mathrm{B}_j = \left(\beta_{j,m}\right)$. Define the variables: $y_{j,t} = \left(R_{j,t} - R_{f,t-1}\right) - \alpha_j$, and the Tx1 matrix Z with t[th] row given by the factor returns $z_t = \left(R_{m,t} - R_{f,t-1}\right)$ for t=1, … , T. Then, combining the prior with the likelihood function

$$L(B_j) \propto (\sigma_{j,\varepsilon}^2)^{-T/2} \exp\left[-\frac{1}{2\sigma_{j,\varepsilon}^2}(Y_j - Z\,\mathrm{B}_j)'(Y_j - Z\,\mathrm{B}_j)\right] \tag{4}$$

we obtain a normal full conditional posterior distribution with mean and variance matrix:

$$EB_j = W_j \left(\frac{\sum_{t=1}^{T} y_{j,t} z_t}{\sum_{t=1}^{T} z_t z_t}\right) + \left(1 - W_j\right) m_B \quad \text{and} \quad VB_j = (1 - W_j)\Omega_B \tag{5}$$

where $W_j = \left(\Omega_B \sum_{t=1}^{T} z_t z_t + \sigma_{j,\varepsilon}^2\right)^{-1} \Omega_B \sum_{t=1}^{T} z_t z_t$.

Given (α_j, B_j), the likelihood for the firm *j* variance follows immediately from (4). We use Bayes' theorem to derive the full conditional posterior for the variance $\sigma_{j,\varepsilon}^2$ as

$$IG(C_1+\frac{T}{2},\, C_2+\frac{1}{2}[\,(Y_j-Z\,B_j)'(Y_j-Z\,B_j)+(B_j-m_B)'\,\Omega_B^{-1}(B_j-m_B)]\,) \quad (6)$$

Given J securities, the cross sectional sample mean is $E\sigma_\varepsilon^2=\frac{1}{J}\sum_{j=1}^{J}\sigma_{j,\varepsilon}^2$ and the sample variance is $V\sigma_\varepsilon^2=\frac{1}{J-1}\sum_{j=1}^{J}\left(\sigma_{j,\varepsilon}^2-E\sigma_\varepsilon^2\right)^2$. We can then obtain $C_1=\left(E\sigma_\varepsilon^2\right)^2\left(V\sigma_\varepsilon^2\right)^{-1}+2$ and $C_2=E\sigma_\varepsilon^2\left(C_1-1\right)$.

Updating the hyperparameters at kth iteration:

We update hyper parameters: m_B, Ω_B, m_α and υ_α^2 using cross sectional information at each k^{th} iteration given $j = 1, \ldots, J$ securities. To complete the model specification, we note that the likelihood function for the factor loadings sample $(B_1, B_2, \ldots, B_J)$, generated at the k^{th} iteration of the Markov Chain, is proportional to:

$$L(B_1,\ldots,B_J)\propto \exp\left[\frac{1}{2}\sum_{j=1}^{J}(B_j-m_B)'(\Omega_B)^{-1}(B_j-m_B)\right] \quad (7)$$

We assume a vague prior of $N\left(0,10^2\right)$ for m_B. In turn, the vague prior implies a normal posterior distribution with mean and variance:

$$Em_B=W_B\bar{B} \text{ and } Vm_B=\left(1-W_B\right)10^2 \quad (8)$$

where $W_B = \dfrac{10^2 J}{\Omega_B + 10^2 J}$ and $\bar{B}$ is the cross-sectional sample mean of B_j. For Ω_B, we use an uninformative inverse Gamma prior $IG\left(C_{3,} C_4\right)$ with $C_3 \approx 0.01$ and $C_4 \approx 0.01$. The values of Ω_B then may be drawn from the Inverted Gamma posterior distribution: $IG(0.01 + \dfrac{J}{2}, \, 0.01 + \dfrac{1}{2}\sum_{j=1}^{J}(B_j - m_B)'(B_j - m_B)\ )$.

The hyper parameter m_α is assumed to have a diffuse prior $N\left(0, 10^2\right)$. This leads to a normal distribution with

$$Em_\alpha = W_\alpha \bar{\alpha} \quad \text{and} \quad Vm_\alpha = \left(1 - W_\alpha\right)10^2 \tag{9}$$

where $W_\alpha = \dfrac{J\left(10^2\right)}{J\left(10^2\right) + \upsilon_\alpha^2}$ and $\bar{\alpha}$ is the sample mean of α_j across our J securities. Finally, we update υ_α^2. Using uninformative inverse Gamma prior, we then obtain

$$\upsilon_{j,\alpha}^2 \sim IG\left(\frac{1}{2}J + 0.01, \frac{1}{2}\sum_{j=1}^{J}\left(\alpha_j - m_\alpha\right)^2 + 0.01\right) \tag{10}$$

given the cross sectional information from J firms.

CoEC improvements in the Bayesian Hierarchical formulation above is expected to come from cross-sectional information, affecting estimated risk loading (β) and pricing error (α). The additional risk loading and pricing error information is formally incorporated via equation (8), (9) and (10). To obtain CoEC, we use long run mean as factors to facilitate comparison with the benchmark OLS CoEC estimates – identical factors are incorporated to the CoEC formulations the same way across our null and alternative models to facilitate comparisons. Given the conditional posteriors for the parameters above, we use the Markov Chain Monte Carlo (MCMC) framework to estimate the parameters and then obtain the Bayesian Hierarchical CAPM CoEC for each firm *j* as follows:

Algorithm 1: Bayesian Hierarchical CAPM.

1. Draw β from a Normal distribution with mean and variance given by Equation (5).

2. For the market risk premium and risk free rate, we use the mean $\bar{R}_m - \bar{R}_f$ = 0.0794 and $\bar{R}_f$ = 0.0363.

3. Given these values, compute the Bayesian Hierarchical CoEC: $CoEC = R_f + \beta_m (R_m - R_f)$

4. Go back to step 1 and repeat K times.

For the initial iteration (k=0) we set the three unknown parameters Θ equal to the ordinary least squares estimates. For each firm, we generate K=2,000 Bayes estimates of the parameters and CoEC. To allow proper mixing we discard the first 1,000 estimates; also, we keep every fifth observation. The remaining 200 observations represent the final sample from the posterior distribution of the parameters and CoEC implied by the CAPM. We use the mean of these observations as the final estimate of each parameter and the Bayesian Hierarchical CoEC.

A specification of CoEC uncertainty however includes factor imprecision. We utilize Fama and French (1997) CoEC uncertainty specification in our study. First, note that the risk premium $\beta\left(ER_m - R_f\right)$ can be approximated by $\hat{\beta}\left(\bar{R}_m - \bar{R}_f\right)$ where $\hat{\beta}$ is the estimated loading and $\bar{R}_m - \bar{R}_f$ is the long run mean of market risk premium factor. In addition, observe that possible CoEC estimation error, symbolized in our study by $e\left(CoEC\right)$, can be obtained as the following

$$e(CoEC) = \hat{\beta}\left(\bar{R}_m - \bar{R}_f\right) - \beta\left(ER_m - R_f\right) = \beta e\left(ER_m - R_f\right) + e\left(\beta\right)\left(ER_m - R_f\right) + e\left(\beta\right)e\left(ER_m - R_f\right)$$

assuming that risk free rate is constant. The symbols $e\left(ER_m - R_f\right)$ and $e\left(\beta\right)$ specifies errors associated with market risk premium factor and market risk premium loading. The dispersion of CoEC is then specified as the standard deviation of CoEC estimation error. Assuming an uncorrelated $e\left(ER_m - R_f\right)$ and $e\left(\beta\right)$, we can obtain the squared CoEC dispersion specification

$$Var\left(e(CoEC)\right) = \beta^2 Var\left(e\left(ER_m - R_f\right)\right) + \left(ER_m - R_f\right)^2 Var\left(e\left(\beta\right)\right) + Var\left(e\left(\beta\right)\right) Var\left(e\left(ER_m - R_f\right)\right)$$

similar to Fama and French (1997). To make the above variance operational, we apply Bayesian Hierarchical beta and the long run average of market excess return respectively, for the true beta β and true market risk premium $ER_m - R_f$. $Var\left(e\left(\beta\right)\right)$ is substituted by the variance of the 200 Bayesian Hierarchical estimate of beta while $Var\left(e\left(ER_m - R_f\right)\right)$ is substituted by the square of market risk premium standard error. Market risk premium standard error is defined as the standard deviation of long run market risk premium data divided by the square root of the number of data.

As our first benchmark, we use full sample OLS to obtain risk loadings estimate. Given the risk loadings and using long run average factor, we obtain OLS CoEC estimates. CoEC dispersion is calculated using Fama and French (1997) formula as before with OLS beta standing-in for true beta. Squared OLS beta standard error is used to proxy $Var\left(e\left(\beta\right)\right)$. We also obtain a comparable CoEC model to OLS within a Bayesian setting as a second benchmark model: Bayesian CoEC with vague prior. The conditional posterior distribution of α_j, B_j and $\sigma^2_{j,\varepsilon}$ are identical to (3), (5) and (6). We assume vague priors to the hyper parameters: $\alpha_j \sim N\left(0,100\right)$, $B_j \sim N\left(0,100\right)$ and $\sigma^2_{j,\varepsilon} \sim IG\left(0.01,0.01\right)$. We therefore have $m_\alpha = m_B = 0 =$ 0, $\upsilon^2_\alpha = \Omega_B = 100$ and $C_1 = C_2 = 0.01$; the hyper parameters are not updated at each iteration in the Bayesian CAPM with vague prior model. As before, MCMC framework is used to obtain Bayesian with vague CoEC for each firm *j* using Algorithm 1. Finally, the dispersion of Bayesian with vague prior CoEC is calculated as before. We use the corresponding Bayesian with uninformative prior loading to proxy for the true beta. The variance of the Bayesian beta estimate is used to proxy for $Var\left(e\left(\beta\right)\right)$. The surrogate for true market risk premium factor $ER_m - R_f$ and $Var\left(e\left(ER_m - R_f\right)\right)$ in the benchmark models are identical to the ones used in the alternative model dispersion – facilitating comparisons of CoEC imprecision across model.

Table 3 presents the benchmark factor loading ($\beta_{j,m}$), loading dispersion (Std($\beta_{j,m}$)), annual CAPM CoEC and uncertainty of annual CAPM CoEC for each firm *j*. CoEC estimates and dispersions are in percent. Panel A reports estimates obtained via Bayesian with vague prior while Panel B reports estimates obtained via OLS. As shown, the two null models obtain similar results. Over 60 firms, we obtain similar loading and loading dispersion: 0.574 and 0.09 respectively. Looking at each firm, loadings of the two benchmark models are generally similar and are all statistically different from zero for 60 all firms. Highest Posterior Density Interval at 1%, 5% and 10% statistical significance for loadings estimated using Bayesian framework and t-statistics for loadings estimated using OLS for each firm are not reported to conserve space and are available from the author. On average, Bayesian with uninformative prior CoEC is 8.186% per year with a dispersion of 1.541% per year. Annual OLS CoEC averages 8.182% with dispersion of 1.531% over our 60-firm sample. The benchmark single-factor CoEC agrees with Arifin (2013) estimates. As shown, there is a considerable variation in the single-factor CoEC estimates. For example, Panel A reports that firm MNRTA obtains CAPM CoEC of 5.684% per year while firm FCH reports annual CoEC of 16.290%. On the other hand, Panel B reports that firm MNRTA obtains annual single-factor CoEC of 5.636% while firm FCH reports annual CoEC of 16.275%.

Taking advantage of CoEC variations among our 60 firms in a Bayesian Hierarchical setting, we report the factor loadings, uncertainty associated with factor loading, single-factor CoEC and CoEC uncertainties in Table 4. The average beta is 0.559 with dispersion of 0.082. The average Bayesian Hierarchical beta is very close to those of the null model – Bayesian Hierarchical average beta is only about 2% to 3% lower than benchmarks. In addition, Bayesian Hierarchical setting allows us to lower the average OLS beta uncertainty by 15.11% (= beta dispersion under Bayesian Hierarchical model minus beta dispersion under OLS model divided by beta dispersion of OLS = [0.082 – 0.097]/0.097). Compared to the benchmark of Bayesian with vague prior, the average Bayesian Hierarchical beta over 60 firms lowers uncertainty by 16.54%. Improvements in the average CoEC is modest however. The average annual Bayesian Hierarchical CoEC is 8.063% -- approximately 1.5% lower than the null CoECs. In addition, looking at the average uncertainty surrounding the CoEC estimate, we find that using Bayesian Hierarchical framework lowers the CAPM CoEC average dispersion by 6.17% against OLS and by 6.76% against Bayesian with uninformative prior.

Given a single factor, the effect of "borrowed" information for a firm's CoEC estimate is readily understood. Looking at firm-by-firm estimates, through the effect the "revised" loadings, Table 4 shows that annual Bayesian Hierarchical CoEC estimate can be different than those of the null model – this is expected. Due to the additional cross-sectional information, we expect a firm with a lower-than-average benchmark CoEC to end up with a higher alternative CoEC estimate and vice versa. In other words, if other firms exposed to the a particular systematic factor have a high benchmark discount rate, we would expect a firm exposed to the same systematic factor with a low benchmark discount rate to have a higher "revised" discount factor and vice versa. We observe this phenomenon in Table 4. For example, firm RPI has a benchmark annual CoEC estimates of 5.923% and 6.031%. In contrast, average annual benchmark CoEC is 8.186% and 8.182%. Annual Bayesian Hierarchical CoEC for firm RPI is 6.739% as shown in Table 4. The annual CoEC of firm RPI is "revised" upward, reflecting the additional information contained in the cross-section. As a second example, firm FCH has a benchmark annual CoEC of 16.290% and 16.275%. The annual Bayesian Hierarchical CoEC for firm FCH is 12.010%. The yearly CoEC for firm FCH is "revised" downward.

Firm-by-firm estimates show that Bayesian Hierarchical CoEC has lower uncertainties for 47 out of 60 firms against the null of Bayesian with vague prior CoEC. When we compare against the benchmark OLS CoEC, Table 4 shows that the firm-by-firm uncertainties of Bayesian Hierarchical CoEC decreases for 41 firms. We must stress however that the increased CoEC accuracy afforded by Bayesian Hierarchical framework at the firm-level is small for the majority of our sample – for example, firm ESS shows an improved accuracy of only 1% compared to the dispersions of benchmark CoEC. Furthermore, the improvements in accuracy are inconsistent. Using the alternative model on firm HCN *lowers* the CoEC dispersion by 1% when we compare against Bayesian with uninformative prior model. At the same time, firm HCN has a 3% *higher* alternative CoEC dispersion when we use OLS CoEC as our benchmark. In sum, our results indicate that utilizing Bayesian Hierarchical framework does not appear to decisively improve the accuracy of single-factor CoEC estimates at the firm level.

Next, we analyze the three-factor CoEC. For each firm *j*, the Fama and French (1993) three-factor CoEC is

$$R_{j,t} - R_{f,t-1} = \alpha_j + \beta_{j,m}(R_{m,t} - R_{f,t-1}) + \beta_{j,smb} R_{smb,t} + \beta_{j,hml} R_{hml,t} + \varepsilon_{j,t} \qquad (11)$$

For each firm, the three factor model contains five unknown parameters: $\Theta_j = \left(\alpha_j, \sigma^2_{j,\varepsilon}, \beta_{j,m}, \beta_{j,smb}, \beta_{j,hml}\right)$ that may be summarized as $\Theta_j = \left(\alpha_j, \sigma^2_{j,\varepsilon}, B_j\right)$ with $\mathrm{B}_j = \left(\beta_{j,m}, \beta_{j,smb}, \beta_{j,hml}\right)'$. Additionally, we define the hyper parameters for our models as $\alpha_j \sim N\left(m_\alpha, \upsilon^2_\alpha\right)$, $B_j \sim N\left(m_B, \Omega_B\right)$ and $\sigma^2_{j,\varepsilon} \sim IG\left(C_1, C_2\right)$.

For each firm *j*, we obtain posterior distribution of α_j as follows. To obtain the likelihood function, conditional on beta, define the unexpected return variable: $y_{j,t} = R_{j,t} - R_{f,t-1} - [\beta_{j,m}(R_{m,t} - R_{f,t-1}) + \beta_{j,smb} R_{smb,t} + \beta_{j,hml} R_{hml,t}]$ for the three-factor model for time periods t=1, … , T. The moments of pricing error α (given the data and the remaining parameters) follow (3).

The second parameter block to be estimated is the factor loading (column) vector $\mathrm{B}_j = \left(\beta_{j,m}, \beta_{j,smb}, \beta_{j,hml}\right)'$. Define the variables: $y_{j,t} = (R_{j,t} - R_{f,t-1}) - \alpha_j$, and the Tx3 matrix Z with tth row given by the factor returns $z_{j,t} = (R_{m,t} - R_{f,t-1}, R_{smb,t}, R_{hml,t})$ for t=1, … , T. Given likelihood function (A-4), we obtain a multivariate normal full conditional posterior distribution with mean and variance matrix:

$$EB_j = \left(Z'Z + \sigma^2_{j,\varepsilon}\left(\Omega_B\right)^{-1}\right)^{-1}\left(Z'Y_j - \sigma^2_{j,\varepsilon}\left(\Omega_B\right)^{-1} m_B\right) \qquad (12\text{-A})$$

and

$$VB_j = \sigma^2_{j,\varepsilon}\left(Z'Z + \sigma^2_{j,\varepsilon}\left(\Omega_B\right)^{-1}\right)^{-1} \qquad (12\text{-B})$$

Given $\left(\alpha_j, B_j\right)$ then, full conditional posterior for the variance $\sigma^2_{j,\varepsilon}$ is identical to (6).

Updating the hyperparameters at kth iteration:

We update hyper parameters: m_B, Ω_B, m_α and υ_α^2 using cross sectional information at each kth iteration given *j = 1, ..., J* securities as before. We note that the likelihood function for the factor loadings sample $(B_1, B_2, \ldots, B_J)$ is similar to (7). We assume a vague prior of $N\left(0, 10^2 I_3\right)$ for m_B where I_3 is 3x3 identity matrix; this implies a multivariate normal posterior distribution with mean and variance:

$$Em_B = \left(10^2 I_3 + \sum_{j=1}^{J} (\Omega_B)^{-1}\right)^{-1} \left((\Omega_B)^{-1} \sum_{j=1}^{J} B_j\right) \text{ and } Vm_B = (10^2 I_3)^{-1} + \sum_{j=1}^{J} (\Omega_B)^{-1} \tag{13}$$

For the variance matrix Ω_B, we assume a Geisser and Cornfield (1963) prior. The resulting conditional posterior is an inverse Wishart: $IW\left(\Psi'\Psi, J\right)$ where Ψ is a stacked $B_j^{'} - m_B^{'}$ matrix with size Jx3. The hyper parameter m_α is assumed to have a diffuse prior $N\left(0, 10^2\right)$ resulting in identical posterior distribution as (9). Additionally, we update $\upsilon_{j,\alpha}^2$ using (10).

CoEC improvements are expected to come from factor loadings (B) – including interactions across loadings – and estimates of pricing error (α) – equation (13), (9) and (10). To facilitate comparison across three-factor models therefore, we obtain CoEC using long run average factors. Given the conditional posterior distributions, we again use MCMC to estimate the parameters and obtain Bayesian Hierarchical three-factor CoEC for each firm *j*. The algorithm that we follow is:

Algorithm 2: Bayesian Hierarchical Fama and French (1993).

1. *Draw β from a Multivariate Normal distribution with mean and variance given by Equation (12).*

2. For the risk free rate, we use the long run mean $\bar{R}_f$ *= 0.0363. For factors, we use the long run mean: the market risk premium factor mean* $\bar{R}_m - \bar{R}_f$ *= 0.0794, the smb factor premium mean* $\bar{R}_{smb}$ *= 0.0366, the hml factor premium mean* $\bar{R}_{hml}$ *= 0.0473.*

3. Obtain Bayesian Hierarchical CoEC: $CoEC = R_f + (R_m - R_f)\beta_m + R_{smb}\beta_{smb} + R_{hml}\beta_{hml}$

4. Go back to step 1 and repeat K times.

We run the simulation for 2,000 iterations and keep every fifth observation after the initial 1,000 burn-in period. A final sample of 200 CoEC observations is then obtained for each firm *j*. As the final estimate and uncertainty of CoEC we use the average of the resulting 200 CoEC. Measure of three-factor discount rate uncertainty follows Fama and French (1997) specification. Define a column vector of the true factor loadings $B = (\beta_m, \beta_{smb}, \beta_{hml})'$ and risk premiums $RP = \left(ER_m - R_f, ER_{smb}, ER_{hml}\right)'$ that can be approximated by estimated risk loadings $\hat{B} = \left(\hat{\beta}_m, \hat{\beta}_{smb}, \hat{\beta}_{hml}\right)'$ and risk premium's long run mean $RP = \left(\bar{R}_m - \bar{R}_f, \bar{R}_{smb}, \bar{R}_{hml}\right)'$. The error associated with three-factor CoEC following Fama and French (1997) specification is then $e(CoEC) = e(B)' RP + B'\left(e(RP)\right) + e(B)' e(RP)$ where $e(B)$ is a column vector of errors associated with the factor loadings and $e(RP)$ is the column vector of errors associated with the factors. The squared uncertainty of the three-factor CoEC as per Fama and French (1997) is then $Var\left(e(CoEC)\right) = RP'Var\left(e(B)'\right)RP + B'Var\left(e(RP)\right)B + Var\left(e(B)'\right)Var\left(e(RP)\right)$, assuming that

$e(B)$ and $e(RP)$ is uncorrelated. To make the variance operational we apply the estimated beta vector and the average factor returns, respectively, for the true factor loadings and the true risk premiums. $Var\left(e(B)^{'}\right)$ is obtained from the variance of the estimated Bayesian Hierarchical loadings, and $Var\left(e(RP)\right)$ is obtained as squared standard error of the factors. Standard error of the factors is defined as the standard deviation divided by the square root of the number of data.

We have two benchmark three-factor CoEC models. First, we obtain full sample OLS CoEC. Uncertainty of the resulting OLS discount factor follows the alternative CoEC uncertainty with $e(B)$ substituted by the resulting OLS loadings and $Var\left(e(B)^{'}\right)$ approximated by the standard errors of the OLS regression slopes. Second, we estimate three-factor Bayesian CoEC with vague prior: $\alpha_j \sim N(0,100)$, $B_j \sim N(0_{3x1}, 100I_3)$ and $\sigma^2_{j,\varepsilon} \sim IG(0.01, 0.01)$. The conditional posterior distribution of α_j, B_j and $\sigma^2_{j,\varepsilon}$ are set to (3), (12) and (6). CoEC for each firm *j* were obtained via MCMC as per Algorithm 2. To obtain CoEC uncertainty, $e(B)$ is replaced by the mean of the 200 Bayesian loadings and $Var\left(e(B)^{'}\right)$ is substituted by the variance of the estimated Bayesian loadings.

Table 5 presents the results of Bayesian with vague prior specifications while Table 6 presents the results of full sample OLS. At each table, we present the market risk premium ($R_m - R_f$ **)** loadings ($\beta_{j,m}$), SMB loading ($\beta_{j,smb}$) , HML loading ($\beta_{j,hml}$) and their associated uncertainties (Std ($\beta_{j,m}$), Std ($\beta_{j,smb}$), and Std ($\beta_{j,hml}$)) in addition to CoEC and CoEC uncertainty. Estimates of CoECs and their uncertainties are in yearly percentage. Table 5 and 6 shows similar estimates on average. The average loadings for Bayesian with uninformative prior model is 0.568, 0.368 and

0.665 for market risk premium, SMB and HML factors respectively. OLS obtain average market risk premium, SMB and HML loadings of 0.568, 0.369 and 0.666 respectively. Average dispersions of the two benchmark models are also similar. Looking at the firm-level results, we find comparable loadings across under the null models. Furthermore, both market risk premium and HML loadings are statistically different from zero at 5% level for all firms under both null models. SMB loadings are indistinguishable from zero at 5% level for 12 firms under OLS model and for 13 firms under Bayesian with uninformative prior. Highest Posterior Density Interval at 1%, 5% and 10% significance for loadings estimated using Bayesian with uninformative prior and t-statistics for loadings estimated via OLS are available from the author.
On average, we find Bayesian with vague prior CoEC to be 12.628% per year while OLS CoEC is 12.638%. The average annual uncertainties surrounding the CoECs are 2.096% and 2.082%. Firm-by-firm CoECs and their associated dispersion are comparable under the null models. These results are comparable to Arifin (2013). Moreover, there is a wide variation in firm-level CoEC estimates, also found in Arifin (2013). Estimating CoEC as Bayesian with vague prior specification, firm MNRTA obtains 7.850% per year while firm FCH obtains 26.724% per year. Using OLS, firm MNRTA obtains Fama and French (1993) CoEC of 7.850% per year while firm FCH obtains 26.974% per year.

Bayesian Hierarchical three-factor results are presented in Table 7. The average loadings appear to be lower under the alternative model. The average loadings are 0.447, 0.212 and 0.425 for market risk premium, SMB and HML loadings respectively when we “borrow” information from the cross-section. In contrast, the average loadings utilizing Bayesian with vague prior are 0.568, 0.368 and 0.665 for market risk premium, SMB and HML factors respectively. Using OLS, we obtain 0.568, 0.369 and 0.666 as average market risk premium, SMB and HML loadings. The average uncertainties of Bayesian Hierarchical loadings are higher than those of the benchmark loadings however. Bayesian Hierarchical loadings have average dispersion of 0.130, 0.174 and 0.184 for market risk premium, SMB and HML loadings correspondingly. When we use OLS, the average standard errors are 0.093, 0.126 and 0.126 and when we use Bayes with vague prior, the average dispersions are 0.094, 0.129 and 0.130 for market risk premium, SMB and HML loadings respectively. Overall, Bayesian Hierarchical CoEC averages 9.966% per year – a decrease of about 21% over the average benchmark CoECs. The average uncertainty of alternative CoEC is 2.027% per year – a decrease of only approximately 2.5% over the benchmark CoECs uncertainties. The reduction in average CoEC and discount rate uncertainty within the Bayesian Hierarchical model are caused by the "revised" loadings due to cross-sectional information at the firm level. We turn to firm-level results next.

Unlike a single-factor model, the effects of "borrowed" information in the loadings are harder to identify in a multi-factor environment where the factors affect one another. At the firm-level, we generally see a drop in the loadings for all firms – affecting the average loadings and thus lowering the average Bayesian Hierarchical CoEC. For example, firm HR has Bayesian Hierarchical loadings of 0.477, 0.174 and 0.511 for market risk premium, SMB and HML factors correspondingly. Under OLS, firm HR has loadings of 0.595, 0.317 and 0.778 for market risk premium, SMB and HML factors. Under Bayesian with vague prior model, the market risk premium, SMB and HML loadings for firm HR are 0.593, 0.318 and 0.777 respectively. In addition, loadings for market risk premium are indistinguishable from zero for 4 firms at 5% level while HML loadings are indistinguishable from zero for 11 firms at 5% level. SMB loadings are indistinguishable from zero for 54 firms at 5% level. Statistically, incorporating cross-sectional information appears to weaken size and book-to-market effect in our sample. Equity REITs sensitivities to SMB and HML factors are not well understood. Using index data from 1975 to 1997, Chiang and Lee (2002) find that Equity REITs' sensitivity to the size effect is unstable over time – there are periods where Equity REITs shows no sensitivity to the size effect. On the contrary, Equity REITs consistently shows sensitivity to the value (book-to-market) effect. Overall, Chiang and Lee (2002) conclude that Equity REITs performs as small cap stocks with a stronger book-to-market effect compared to the size effect after the 1990s. In our current framework, we do not take into account time variation in SMB and HML effects. It is possible that the inclusion of cross-sectional information negates the size and book-to-market effects at the firm-level. We leave exploration of such issues to future studies.

We also see an increase in loadings dispersions for all firms. Firm HR's loadings dispersions under the alternative discount rate model are 0.136, 0.172 and 0.175 for market risk premium, SMB and HML loadings. At the same time, firm HR's loadings dispersions under OLS are 0.093, 0.127 and 0.127 for market risk, SMB and HML loadings. Firm HR's loadings dispersion under Bayesian with vague prior model are 0.100, 0.134 and 0.130 for market risk, SMB and HML loadings. Thus, while "borrowing" information appears to lower firm-level CoEC estimates, the effect on CoEC uncertainties is less clear – CoEC uncertainties takes into account both loadings imprecision and estimates. Utilizing Bayesian Hierarchical framework lowers the CoEC dispersion in 54 firms when we use Bayesian with uninformative prior as our benchmark. With OLS as our benchmark, using Bayesian Hierarchical specification lowers CoEC dispersion in 53 out of 60 firms. These reductions in firm-level CoEC uncertainty are modest however.

Firm HR reports a decrease in CoEC uncertainty of approximately 4.5% against the benchmarks. Bayesian Hierarchical framework does not appear to overwhelmingly help us obtain a more accurate three-factor CoEC.

3.4 Conclusion and Issues for Future Research

We find that Bayesian Hierarchical specification have only a mild effect on Equity REITs CoEC in our sample. In the single-factor CAPM model, "borrowing" information appears to have only a modest effect on average: estimated average CoEC is only about 1.5% lower than the benchmark discount factors and the estimated average CoEC uncertainty is only about 6% lower than the benchmark discount rate uncertainties. Looking at the firm-by-firm results, Bayesian Hierarchical framework lowers CAPM CoEC uncertainty for the majority of our sample – the improvements are however modest and inconsistent. In the Fama and French (1993) three-factor model, specifying a Bayesian Hierarchical framework lowers the average CoEC by about 21% against the benchmarks; average CoEC uncertainty decrease by about 2.5% against the benchmarks. At the firm-level, while using additional cross-sectional informational lowers CoEC uncertainty for the majority for our sample, the improvements are also fairly small and inconsistent.

What are the implications of our finding? If our goal is to improve in-sample Equity REITs CoEC accuracy, cross-sectional information in our sample appears to be of limited use. In this study, we "borrow" information to revise loadings estimate; we take factors and their uncertainty as given. A number of studies have found that imprecision in factors is more important than loadings imprecision. Using industry data, Fama and French (1997) conclude that factors imprecision has a larger effect on CoEC accuracy than loadings imprecision. Pastor and Stambaugh (1999) find that factor imprecision is the biggest contributor to overall uncertainty of the expected excess return. Pastor and Stambaugh (1999) however also suggest that loadings uncertainty is nearly as important for an individual stock as the uncertainties surrounding the factors. It may be possible that factors imprecision dominates risk loadings uncertainty in Equity REITs CoEC.

Second, we find that including cross-sectional information appear to statistically weaken size and book-to-market effect within the three-factor CoEC at the firm-level i.e. Bayesian Hierarchical SMB and HML loadings are not statistically different from zero for a large proportion of our firm-sample compared to loadings of the benchmark models. Sources of sensitivities to size and book-to-market factors remain a subject of considerable debate – for Equity REITs, size and book-to-market risk sensitivities are not well understood. Additional studies are needed to clarify the sources of SMB and HML sensitivities in the context of Equity REITs.

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Table 1 - Summary statistics of our 60-firm sample.

				Firm Returns				Volume	
Ticker	Firm Name	REIT Type	Mkt Val	Ave	Std	Skew	Kurt	Ave	Std
EPR	Entertainment Properties Trusts	Diversified	2,042	0.211	1.155	-0.110	8.648	1.559	3.609
WRE	Washington Real Estate Investment Trust	Diversified	1,807	0.112	0.781	-0.661	5.732	2.409	5.251
LXP	Lexington Realty Trust	Diversified	1,156	0.129	1.251	-0.029	15.270	3.687	8.439
CUZ	Cousins Properties Inc.	Diversified	665	0.041	1.106	-0.541	7.628	2.766	6.106
IRETS	Investors Real Estate Trusts	Diversified	610	0.077	0.545	0.006	3.594	0.922	2.136
HCP	HCP, Inc.	Health Care	16,928	0.188	0.965	-0.084	6.054	10.953	28.458
HCN	Health Care REIT, Inc.	Health Care	10,349	0.163	0.810	-0.095	3.411	5.588	13.883
OHI	Omega Healthcare Investors, Inc.	Health Care	1,996	0.148	1.666	0.180	5.576	3.058	6.756
HR	Healthcare Realty Trust	Health Care	1,447	0.113	0.936	-0.350	4.673	2.850	5.844
LTC	LTC Properties, Inc.	Health Care	936	0.168	1.067	-0.309	4.499	0.985	1.374
PLD	ProLogis Trust	Industrial	13,129	0.242	2.880	7.948	91.394	22.734	65.241
EGP	EastGroup Properties, Inc.	Industrial	1,177	0.154	0.806	-0.780	6.293	0.853	1.651
FR	First Industrial Realty Trust, Inc.	Industrial	886	0.102	1.512	-0.745	12.431	3.448	6.913
MNRTA	Monmouth Real Estate Investment Corp.	Industrial	359	0.139	0.624	0.704	6.698	0.262	0.549
HPT	Hospitality Properties Trust	Lodging	2,839	0.137	1.051	-0.687	9.247	4.999	11.261
LHO	LaSalle Hotel Properties	Lodging	2,028	0.224	1.687	1.898	21.447	2.852	6.839
FCH	FelCor Lodging Trust Incorporated	Lodging	380	0.075	2.117	0.724	7.139	4.262	8.044
EQR	Equity Residential Properties Trust	Multi-family	16,917	0.166	0.871	-0.566	4.690	13.660	28.908
AVB	AvalonBay Communities Inc.	Multi-family	12,418	0.184	0.869	-0.513	4.504	4.940	11.692
ESS	Essex Property Trust, Inc.	Multi-family	4,795	0.195	0.830	-0.195	3.581	1.880	4.251
CPT	Camden Property Trust	Multi-family	4,441	0.164	0.956	-0.525	5.365	3.456	7.333
BRE	BRE Properties, Inc.	Multi-family	3,801	0.144	0.897	-0.522	5.469	3.281	7.025
HME	Home Properties Inc.	Multi-family	2,779	0.153	0.809	-0.876	6.298	1.073	1.425
AIV	Apartment Investment & Management Co.	Multi-family	2,770	0.120	1.242	-0.966	9.268	7.875	17.651
MAA	Mid-America Apartment Communities, Inc.	Multi-family	2,366	0.173	0.786	-0.663	5.649	1.418	3.088
PPS	Post Properties, Inc.	Multi-family	2,268	0.111	1.000	-0.449	4.840	3.225	6.054
CLP	Colonial Properties Trust	Multi-family	1,823	0.160	1.436	2.270	28.998	2.695	5.764
AEC	Associated Estates Realty Corporation	Multi-family	675	0.173	1.171	-0.115	5.968	0.910	1.699

Table 1 - Continued

Ticker	Firm Name	REIT Type	Mkt Val	Firm Returns Ave	Std	Skew	Kurt	Volume Ave	Std
BXP	Boston Properties, Inc.	Office	14,726	0.179	0.906	0.183	8.334	6.819	15.618
SLG	SL Green Realty Corp.	Office	5,741	0.219	1.451	-0.004	10.921	4.958	13.780
ARE	Alexandria Real Estate Equities Inc.	Office	4,273	0.150	1.060	-0.178	12.882	2.132	5.512
DRE	Duke-Weeks Realty Corporation	Office	3,048	0.086	1.294	1.675	20.914	10.020	23.984
CLI	Mack-Cali Realty Corporation	Office	2,326	0.097	0.976	0.394	6.969	4.098	8.489
KRC	Kilroy Realty Corporation	Office	2,226	0.139	1.009	-0.553	5.204	2.436	4.959
HIW	Highwoods Properties, Inc.	Office	2,153	0.116	0.905	-0.629	4.208	3.868	7.145
OFC	Corporate Office Properties Trust	Office	1,530	0.170	0.859	-0.486	4.051	2.589	6.138
BDN	Brandywine Realty Trust	Office	1,288	0.131	1.737	3.453	33.582	5.623	13.508
PKY	Parkway Properties, Inc.	Office	217	0.037	1.204	-0.752	9.316	0.618	1.138
SPG	Simon Property Group, Inc.	Retail	37,888	0.212	1.010	0.165	10.412	11.967	30.576
MAC	Macerich Company, The	Retail	6,676	0.270	2.185	5.651	60.772	5.700	15.220
KIM	Kimco Realty Corporation	Retail	6,609	0.132	1.231	0.586	14.390	15.380	43.703
FRT	Federal Realty Investment Trust	Retail	5,762	0.185	0.801	-1.011	5.946	3.113	6.776
O	Realty Income Corporation	Retail	4,657	0.168	0.689	-0.016	3.620	4.004	10.135
TCO	Taubman Centers, Inc.	Retail	3,597	0.216	1.045	-0.348	7.230	3.750	7.321
REG	Regency Realty Corporation	Retail	3,383	0.142	1.001	-0.208	10.359	3.820	9.496
DDR	Developers Diversified Realty Corporation	Retail	3,371	0.158	1.649	0.785	20.190	10.638	27.935
WRI	Weingarten Realty Investors	Retail	2,637	0.128	1.222	1.209	16.292	4.552	10.771
SKT	Tanger Factory Outlet Centers, Inc.	Retail	2,542	0.222	0.791	0.643	8.192	1.972	4.537
CBL	CBL & Associates Properties, Inc.	Retail	2,329	0.290	2.757	6.818	71.222	5.781	14.883
EQY	Equity One, Inc.	Retail	1,947	0.148	0.799	-0.467	4.128	1.942	4.297
ALX	Alexander's Inc.	Retail	1,889	0.180	1.127	-0.356	5.302	0.164	0.195
GRT	Glimcher Realty Trust	Retail	989	0.177	1.609	0.262	11.188	2.551	5.517
AKR	Acadia Realty Trust	Retail	858	0.192	0.858	0.002	8.345	1.131	2.400
BFS	Saul Centers, Inc.	Retail	683	0.158	0.939	0.448	7.087	0.389	0.571
PEI	Pennsylvania Real Estate Investment Trust	Retail	581	0.172	1.874	2.537	24.900	2.222	5.071
RPT	Ramco-Gershenson Properties Trust	Retail	383	0.136	1.359	0.074	18.018	0.743	1.666

Table 1 – Continued

Ticker	Firm Name	REIT Type	Mkt Val	Firm Returns Ave	Std	Skew	Kurt	Volume Ave	Std
OLP	One Liberty Properties, Inc.	Retail	240	0.184	1.342	1.433	19.076	0.212	0.491
ADC	Agree Realty Corp	Retail	240	0.156	1.082	0.118	7.898	0.250	0.432
RPI	Roberts Realty Investors, Inc.	Retail	13	0.051	1.322	1.282	14.332	0.027	0.054
SSS	Sovran Self Storage, Inc.	Self Storage	1,189	0.145	0.900	-0.259	6.056	0.766	1.356

Notes: Table 1 reports summary statistics of the 60 firms in our sample. The period is January 1999 to December 2011. *REIT Type* denotes property focus. *Mkt Val* stands for market value. The market value is in millions of dollars as of 2011 Q4. We obtain market value as price per share multiplied by the number of shares outstanding obtained from CRSP. Firm returns signify annualized 1-month holding-period-return obtained from CRSP. We annualize returns by multiplying with 12. *Ave, Std, Skew* and *Kurt* signify average, standard deviation, skewness and kurtosis respectively. *Volume* is the monthly trading volume in millions of shares.

Table 2 – Summary statistics of Property Type portfolios, NAREIT index and factors.

Factors	Returns Ave	Std	Std. Err.
R_f	0.036	0.031	0.003
R_m - R_f	0.079	0.208	0.023
SMB	0.037	0.142	0.015
HML	0.047	0.139	0.015

Notes: Table 2 reports summary statistics of annual factors from 1927 to 2011. We obtained factors from Dr. Kenneth French's website. *Ave*, *Std*, *Skew* and *Kurt* denote average, standard deviation, skewness and kurtosis respectively. S*td. Err.* signifies standard deviation divided by square root of the number of data.

Table 3 – CAPM cost of equity capital for our 60-firm sample.

	Panel A: Bayesian with Vague Prior				Panel B: OLS			
Ticker	$\beta_{j,m}$	Std ($\beta_{j,m}$)	CoEC	Std (CoEC)	$\beta_{j,m}$	Std ($\beta_{j,m}$)	CoEC	Std (CoEC)
EPR	0.533	0.094	7.857	1.431	0.536	0.097	7.882	1.453
WRE	0.400	0.084	6.799	1.140	0.397	0.080	6.782	1.115
LXP	0.486	0.098	7.485	1.364	0.479	0.095	7.429	1.339
CUZ	0.565	0.103	8.114	1.533	0.566	0.100	8.119	1.522
IRETS	0.223	0.064	5.399	0.727	0.222	0.064	5.389	0.728
HCP	0.427	0.100	7.017	1.271	0.432	0.096	7.060	1.259
HCN	0.400	0.099	6.806	1.222	0.396	0.091	6.770	1.169
OHI	0.781	0.168	9.827	2.243	0.762	0.169	9.677	2.216
HR	0.573	0.098	8.173	1.527	0.575	0.099	8.193	1.535
LTC	0.495	0.125	7.557	1.522	0.495	0.117	7.557	1.475
PLD	0.741	0.090	9.509	1.834	0.730	0.089	9.425	1.807
EGP	0.463	0.079	7.302	1.234	0.466	0.080	7.330	1.246
FR	0.699	0.106	9.176	1.807	0.696	0.114	9.153	1.834
MNRTA	0.259	0.072	5.684	0.834	0.253	0.064	5.636	0.781
HPT	0.598	0.101	8.371	1.588	0.599	0.098	8.380	1.576
LHO	1.005	0.124	11.602	2.491	1.013	0.117	11.668	2.484
FCH	1.596	0.165	16.290	3.857	1.594	0.162	16.275	3.842
EQR	0.611	0.090	8.481	1.571	0.617	0.089	8.528	1.575
AVB	0.638	0.084	8.693	1.601	0.638	0.085	8.688	1.601
ESS	0.539	0.082	7.907	1.396	0.535	0.087	7.877	1.406
CPT	0.642	0.088	8.723	1.622	0.631	0.085	8.633	1.588
BRE	0.504	0.089	7.628	1.355	0.506	0.087	7.647	1.350
HME	0.412	0.079	6.896	1.139	0.402	0.084	6.816	1.141
AIV	0.724	0.106	9.377	1.857	0.725	0.104	9.378	1.850
MAA	0.543	0.076	7.941	1.378	0.538	0.078	7.900	1.377
PPS	0.667	0.090	8.924	1.681	0.672	0.091	8.958	1.695
CLP	0.423	0.074	6.984	1.135	0.427	0.080	7.016	1.168
AEC	0.494	0.108	7.552	1.432	0.493	0.115	7.545	1.463
BXP	0.587	0.087	8.284	1.509	0.589	0.081	8.305	1.489
SLG	0.809	0.098	10.045	1.999	0.801	0.104	9.985	2.004
ARE	0.406	0.074	6.852	1.103	0.414	0.083	6.911	1.157
DRE	0.690	0.093	9.102	1.736	0.681	0.093	9.032	1.719
CLI	0.545	0.095	7.953	1.459	0.546	0.088	7.961	1.433
KRC	0.700	0.109	9.187	1.820	0.713	0.100	9.284	1.811
HIW	0.620	0.105	8.552	1.649	0.616	0.101	8.520	1.622
OFC	0.534	0.102	7.863	1.471	0.533	0.096	7.857	1.441
BDN	0.738	0.108	9.483	1.890	0.725	0.107	9.378	1.862
PKY	0.732	0.119	9.440	1.924	0.735	0.111	9.458	1.896
SPG	0.492	0.093	7.535	1.354	0.496	0.092	7.568	1.357
MAC	0.612	0.102	8.486	1.618	0.606	0.107	8.437	1.630
KIM	0.582	0.087	8.246	1.498	0.582	0.094	8.247	1.529
FRT	0.411	0.085	6.890	1.163	0.408	0.082	6.869	1.145
O	0.349	0.079	6.402	1.026	0.341	0.077	6.336	0.999

Table 3 – Continued

	Panel A: Bayesian with Vague Prior				Panel B: OLS			
Ticker	$\beta_{j,m}$	Std ($\beta_{j,m}$)	CoEC	Std (CoEC)	$\beta_{j,m}$	Std ($\beta_{j,m}$)	CoEC	Std (CoEC)
TCO	0.648	0.081	8.769	1.608	0.649	0.087	8.775	1.633
REG	0.451	0.094	7.208	1.279	0.466	0.085	7.325	1.267
DDR	0.715	0.110	9.301	1.852	0.709	0.110	9.254	1.843
WRI	0.441	0.091	7.129	1.250	0.451	0.088	7.211	1.252
SKT	0.405	0.076	6.843	1.110	0.405	0.082	6.843	1.137
CBL	0.730	0.124	9.424	1.941	0.725	0.118	9.381	1.906
EQY	0.571	0.080	8.158	1.449	0.570	0.081	8.156	1.452
ALX	0.752	0.116	9.595	1.952	0.747	0.111	9.558	1.922
GRT	0.872	0.119	10.546	2.200	0.869	0.127	10.524	2.226
AKR	0.437	0.081	7.096	1.192	0.447	0.078	7.174	1.196
BFS	0.327	0.109	6.227	1.165	0.328	0.100	6.230	1.108
PEI	0.750	0.131	9.583	2.012	0.756	0.118	9.627	1.967
RPT	0.520	0.109	7.758	1.479	0.529	0.105	7.826	1.477
OLP	0.489	0.083	7.508	1.298	0.480	0.074	7.440	1.244
ADC	0.429	0.103	7.034	1.291	0.422	0.097	6.980	1.246
RPI	0.289	0.122	5.923	1.203	0.303	0.111	6.031	1.141
SSS	0.385	0.090	6.683	1.145	0.387	0.090	6.698	1.148
Ave	0.574	0.098	8.186	1.541	0.574	0.097	8.182	1.531

Notes: Table 3 reports factor loading ($\beta_{j,m}$), uncertainty associated with factor loading (*Std*($\beta_{j,m}$)), cost of equity capital (*CoEC*) and uncertainty associated with the cost of equity (*Std (CoEC)*) based on CAPM

$$R_{j,t} - R_{f,t-1} = \alpha_j + \beta_{j,m}(R_{m,t} - R_{f,t-1})$$

of our 60-firm sample. The period is January 1999 to December 2011. CoECs and their associated uncertainties are in percent per year. As factors, we use the annual risk free rate and market risk premium data from 1927 to 2011. The average annual risk free rate is 3.63% while the average and standard error of the annual market risk premium is 7.94% and 2.26%. Panel A reports Bayesian Hierarchical CoEC estimated via Markov Chain Monte Carlo. We generate 200 loadings and CoEC estimates. We report the average factor loading, average CoEC and uncertainty associated with the CoEC. Panel B reports CoEC estimated via full sample ordinary least squares (*OLS*). For both Panel A and B, CoEC uncertainty is estimated as

$$sqrt\left[\beta^2 Var\left(e\left(ER_m - R_f\right)\right) + \left(ER_m - R_f\right)^2 Var\left(e\left(\beta\right)\right) + Var\left(e\left(\beta\right)\right) Var\left(e\left(ER_m - R_f\right)\right)\right]$$

where β is replaced by loading estimates and $ER_m - R_f$ is substituted by market risk premium factor. In addition, $Var\left(e\left(ER_m - R_f\right)\right)$ corresponds to the squared standard error of market risk premium factor. Standard error of the factor is calculated as the time series standard deviation of the factor divided by the square root of the number of data. For Bayesian models, $Var\left(e\left(\beta\right)\right)$ is replaced by the variance of the estimated loadings. For OLS, $Var\left(e\left(\beta\right)\right)$ is replaced by the squared standard error of estimated loading. A*ve* is average of 60 firms.

Table 4 – Hierarchical Bayesian CAPM cost of equity capital for our 60-firm sample.

Ticker	$\beta_{j,m}$	Std ($\beta_{j,m}$)	CoEC	Std (CoEC)
EPR	0.541	0.082	7.922	1.400
WRE	0.438	0.067	7.106	1.136
LXP	0.508	0.083	7.660	1.337
CUZ	0.566	0.082	8.118	1.449
IRETS	0.274	0.056	5.806	0.776
HCP	0.473	0.084	7.386	1.274
HCN	0.440	0.082	7.122	1.205
OHI	0.647	0.101	8.766	1.685
HR	0.569	0.081	8.142	1.447
LTC	0.514	0.097	7.709	1.410
PLD	0.686	0.085	9.073	1.701
EGP	0.492	0.073	7.534	1.265
FR	0.652	0.089	8.803	1.648
MNRTA	0.310	0.065	6.088	0.884
HPT	0.587	0.078	8.286	1.474
LHO	0.845	0.086	10.335	2.038
FCH	1.056	0.148	12.010	2.683
EQR	0.604	0.076	8.420	1.502
AVB	0.614	0.077	8.503	1.526
ESS	0.541	0.079	7.921	1.388
CPT	0.612	0.077	8.488	1.523
BRE	0.518	0.077	7.736	1.332
HME	0.435	0.076	7.080	1.166
AIV	0.670	0.094	8.945	1.700
MAA	0.541	0.073	7.922	1.365
PPS	0.642	0.079	8.726	1.592
CLP	0.462	0.083	7.298	1.248
AEC	0.517	0.082	7.734	1.353
BXP	0.577	0.071	8.211	1.429
SLG	0.717	0.084	9.320	1.765
ARE	0.445	0.080	7.157	1.204
DRE	0.646	0.077	8.753	1.591
CLI	0.549	0.079	7.989	1.404
KRC	0.663	0.083	8.889	1.649
HIW	0.598	0.084	8.376	1.519
OFC	0.545	0.083	7.957	1.409
BDN	0.680	0.090	9.024	1.707
PKY	0.676	0.082	8.993	1.672
SPG	0.509	0.076	7.666	1.310
MAC	0.597	0.091	8.367	1.543
KIM	0.578	0.078	8.214	1.458
FRT	0.441	0.074	7.131	1.170
O	0.386	0.068	6.692	1.038

Table 4 – Continued

Ticker	$\beta_{j,m}$	Std ($\beta_{j,m}$)	CoEC	Std (CoEC)
TCO	0.633	0.077	8.655	1.565
REG	0.491	0.073	7.522	1.263
DDR	0.639	0.084	8.699	1.601
WRI	0.483	0.074	7.465	1.250
SKT	0.438	0.074	7.104	1.163
CBL	0.655	0.101	8.824	1.698
EQY	0.570	0.075	8.154	1.431
ALX	0.674	0.089	8.977	1.692
GRT	0.743	0.095	9.526	1.855
AKR	0.458	0.067	7.267	1.176
BFS	0.392	0.090	6.740	1.157
PEI	0.684	0.096	9.058	1.737
RPT	0.540	0.086	7.912	1.411
OLP	0.491	0.067	7.527	1.239
ADC	0.456	0.081	7.247	1.229
RPI	0.392	0.093	6.739	1.173
SSS	0.426	0.081	7.007	1.171
Ave	0.559	0.082	8.063	1.436

Notes: Table 4 reports factor loading ($\beta_{j,m}$), uncertainty associated with factor loading (*Std*($\beta_{j,m}$)), cost of equity capital (*CoEC*) and uncertainty associated with cost of equity (*Std (CoEC)*) based on the Bayesian Hierarchical CAPM of our 60-firm sample. The period is January 1999 to December 2011. CoEC uncertainty is estimated as

$$sqrt\left[\beta^2 Var\left(e\left(ER_m - R_f\right)\right) + \left(ER_m - R_f\right)^2 Var\left(e\left(\beta\right)\right) + Var\left(e\left(\beta\right)\right) Var\left(e\left(ER_m - R_f\right)\right)\right]$$

where β is replaced by loading estimates and $ER_m - R_f$ is substituted by market risk premium factor. In addition, $Var\left(e\left(ER_m - R_f\right)\right)$ corresponds to the squared standard error of market risk premium factor. Standard error of the factor is calculated as the time series standard deviation of the factor divided by the square root of the number of data. $Var\left(e\left(\beta\right)\right)$ is replaced by the variance of the estimated loadings. CoECs and their associated uncertainty are in percent per year. As factors, we use the annual risk free rate and market risk premium data from 1927 to 2011. The average annual risk free rate is 3.63% while the average and standard error of the annual market risk premium is 7.94% and 2.26%. We generate 200 loadings and CoEC estimates via Markov Chain Monte Carlo framework. *Ave* is average of 60 firms.

Table 5 – Bayesian with Vague Prior Fama and French (1993) three-factor cost of equity for our 60-firm sample.

Ticker	$\beta_{j,m}$	Std ($\beta_{j,m}$)	$\beta_{j,smb}$	Std ($\beta_{j,smb}$)	$\beta_{j,hml}$	Std ($\beta_{j,hml}$)	CoEC	Std (CoEC)
EPR	0.477	0.094	0.582	0.123	0.617	0.119	12.467	2.006
WRE	0.363	0.077	0.510	0.107	0.715	0.106	11.758	1.822
LXP	0.473	0.082	0.456	0.121	0.835	0.125	13.002	2.075
CUZ	0.550	0.100	0.423	0.140	0.676	0.133	12.741	2.108
IRETS	0.209	0.071	0.221	0.086	0.317	0.085	7.593	1.099
HCP	0.477	0.095	0.072	0.128	0.580	0.133	10.421	1.805
HCN	0.392	0.096	0.308	0.126	0.531	0.127	10.381	1.713
OHI	0.742	0.177	0.415	0.225	0.839	0.237	15.005	3.024
HR	0.593	0.100	0.318	0.134	0.777	0.130	13.179	2.191
LTC	0.503	0.120	0.258	0.164	0.570	0.163	11.263	2.065
PLD	0.783	0.088	0.024	0.118	0.482	0.130	12.211	2.197
EGP	0.469	0.083	0.301	0.104	0.588	0.109	11.237	1.752
FR	0.722	0.114	0.351	0.155	0.935	0.138	15.064	2.584
MNRTA	0.254	0.072	0.185	0.096	0.324	0.099	7.850	1.179
HPT	0.614	0.097	0.183	0.141	0.540	0.137	11.725	2.024
LHO	1.003	0.112	0.440	0.150	0.843	0.144	17.188	2.993
FCH	1.634	0.133	0.644	0.200	1.642	0.194	26.724	4.854
EQR	0.651	0.089	0.172	0.112	0.736	0.106	12.910	2.120
AVB	0.658	0.079	0.238	0.110	0.689	0.112	12.983	2.088
ESS	0.526	0.082	0.373	0.106	0.615	0.111	12.078	1.885
CPT	0.637	0.079	0.341	0.119	0.648	0.121	13.003	2.080
BRE	0.508	0.082	0.350	0.123	0.704	0.118	12.277	1.946
HME	0.377	0.090	0.374	0.110	0.542	0.113	10.559	1.671
AIV	0.715	0.099	0.498	0.129	0.897	0.150	15.374	2.557
MAA	0.538	0.087	0.233	0.110	0.368	0.112	10.495	1.714
PPS	0.681	0.083	0.276	0.120	0.656	0.121	13.152	2.144
CLP	0.387	0.079	0.445	0.105	0.547	0.101	10.914	1.663
AEC	0.436	0.111	0.600	0.152	0.720	0.161	12.689	2.203
BXP	0.616	0.081	0.159	0.102	0.540	0.119	11.653	1.902
SLG	0.783	0.091	0.514	0.149	0.775	0.141	15.390	2.556
ARE	0.387	0.085	0.399	0.109	0.566	0.115	10.842	1.697
DRE	0.705	0.094	0.343	0.120	0.764	0.126	14.096	2.318
CLI	0.584	0.081	0.144	0.112	0.659	0.119	11.907	1.943
KRC	0.655	0.098	0.621	0.132	0.684	0.138	14.335	2.362
HIW	0.644	0.100	0.330	0.145	0.838	0.125	13.915	2.319
OFC	0.490	0.082	0.532	0.130	0.660	0.137	12.590	2.022
BDN	0.713	0.097	0.552	0.148	0.922	0.139	15.670	2.595
PKY	0.670	0.103	0.682	0.143	0.799	0.151	15.222	2.544
SPG	0.509	0.084	0.309	0.120	0.706	0.128	12.142	1.950
MAC	0.581	0.104	0.556	0.142	0.801	0.141	14.064	2.339
KIM	0.590	0.100	0.266	0.128	0.589	0.130	12.074	2.027
FRT	0.417	0.085	0.315	0.114	0.636	0.117	11.098	1.756
O	0.307	0.074	0.402	0.095	0.552	0.094	10.151	1.518

Table 5 – Continued

Ticker	$\beta_{j,m}$	Std ($\beta_{j,m}$)	$\beta_{j,smb}$	Std ($\beta_{j,smb}$)	$\beta_{j,hml}$	Std ($\beta_{j,hml}$)	CoEC	Std (CoEC)
TCO	0.667	0.078	0.243	0.121	0.616	0.122	12.728	2.073
REG	0.442	0.090	0.383	0.126	0.538	0.121	11.084	1.779
DDR	0.756	0.102	0.177	0.150	0.774	0.142	13.944	2.435
WRI	0.469	0.093	0.237	0.125	0.647	0.139	11.282	1.879
SKT	0.435	0.083	0.134	0.117	0.441	0.111	9.661	1.563
CBL	0.634	0.108	0.863	0.151	0.942	0.148	16.271	2.748
EQY	0.564	0.079	0.331	0.115	0.520	0.115	11.778	1.861
ALX	0.692	0.106	0.470	0.151	0.515	0.147	13.275	2.288
GRT	0.847	0.117	0.558	0.158	0.909	0.161	16.692	2.873
AKR	0.464	0.076	0.191	0.102	0.460	0.116	10.186	1.600
BFS	0.301	0.101	0.396	0.128	0.571	0.121	10.175	1.704
PEI	0.728	0.105	0.587	0.150	0.941	0.163	16.015	2.706
RPT	0.517	0.104	0.376	0.128	0.584	0.138	11.873	1.988
OLP	0.475	0.073	0.302	0.102	0.488	0.111	10.816	1.658
ADC	0.378	0.101	0.507	0.120	0.611	0.129	11.376	1.879
RPI	0.297	0.115	0.231	0.162	0.400	0.159	8.723	1.698
SSS	0.371	0.083	0.356	0.121	0.542	0.115	10.441	1.644
Ave	0.568	0.094	0.368	0.129	0.665	0.130	12.628	2.096

Notes: Table 5 reports market risk premium loadings ($\beta_{j,m}$), SMB loading ($\beta_{j,smb}$) , HML loading ($\beta_{j,hml}$),and their associated uncertainties (Std ($\beta_{j,m}$), Std ($\beta_{j,smb}$), and Std ($\beta_{j,hml}$)). Furthermore, we report cost of equity capital (*CoEC*) and uncertainty of cost of equity (*Std (CoEC)*). The model is Fama and French (1993) three-factor CoE:

$$R_{j,t} - R_{f,t-1} = \alpha_j + \beta_{j,m}(R_{m,t} - R_{f,t-1}) + \beta_{j,smb}R_{smb,t} + \beta_{j,hml}R_{hml,t}$$

, specified within the Bayesian with uninformative prior framework. We report result of our 60-firm sample. The period is January 1999 to December 2011. CoECs and their associated uncertainties are in percent per year. As factors, we use the annual risk free rate and market risk premium data from 1927 to 2011. The average annual risk free rate is 3.63% while the average and standard error of the annual market risk premium is 7.94% and 2.26%. We generate 200 loadings and CoEC estimates within Markov Chain Monte Carlo (MCMC) framework. The average factor loading and CoEC are reported as $\beta_{j,m}$, $\beta_{j,smb}$, $\beta_{j,hml}$, and *CoEC* estimates. *Std (CoEC)* is estimated as the square root of

$$sqrt\left[RP^{'}Var\left(e(B)^{'}\right)RP + B^{'}Var\left(e(RP)\right)B + Var\left(e(B)^{'}\right)Var\left(e(RP)\right)\right]$$

where RP is replaced by long run mean of factors $RP = \left(\bar{R}_m - \bar{R}_f,\ \bar{R}_{smb},\ \bar{R}_{hml}\right)^{'}$, B is replaced by average Bayesian with vague prior loading estimates $\hat{B} = \left(\hat{\beta}_m,\ \hat{\beta}_{smb},\ \hat{\beta}_{hml}\right)^{'}$. Furthermore, $Var\left(e(RP)\right)$ corresponds to the squared standard error of the factors. Standard errors of the factors are calculated as the time series standard deviation of the factors divided by the square root of the number of data. $Var\left(e(B)^{'}\right)$ is replaced by the variance of the estimated loadings. *Ave* is average of 60 firms.

Table 6 – OLS Fama and French (1993) three-factor cost of equity capital for our 60-firm sample.

Ticker	$\beta_{j,m}$	Std ($\beta_{j,m}$)	$\beta_{j,smb}$	Std ($\beta_{j,smb}$)	$\beta_{j,hml}$	Std ($\beta_{j,hml}$)	CoEC	Std (CoEC)
EPR	0.474	0.092	0.593	0.125	0.626	0.126	12.523	2.025
WRE	0.363	0.071	0.519	0.096	0.719	0.096	11.814	1.792
LXP	0.475	0.087	0.447	0.118	0.829	0.118	12.958	2.070
CUZ	0.547	0.097	0.427	0.131	0.667	0.132	12.688	2.078
IRETS	0.210	0.064	0.226	0.087	0.334	0.087	7.703	1.090
HCP	0.480	0.095	0.085	0.130	0.569	0.130	10.445	1.799
HCN	0.389	0.090	0.305	0.123	0.531	0.123	10.347	1.675
OHI	0.768	0.171	0.407	0.232	0.837	0.232	15.177	3.029
HR	0.595	0.093	0.317	0.127	0.778	0.127	13.190	2.161
LTC	0.503	0.118	0.265	0.160	0.580	0.160	11.336	2.053
PLD	0.781	0.089	0.033	0.121	0.487	0.121	12.252	2.187
EGP	0.467	0.077	0.301	0.105	0.587	0.105	11.216	1.722
FR	0.725	0.106	0.363	0.145	0.944	0.145	15.178	2.580
MNRTA	0.249	0.065	0.187	0.088	0.330	0.088	7.850	1.118
HPT	0.620	0.098	0.196	0.133	0.556	0.133	11.898	2.031
LHO	1.011	0.112	0.441	0.152	0.833	0.152	17.210	3.011
FCH	1.637	0.138	0.673	0.188	1.667	0.188	26.974	4.886
EQR	0.664	0.082	0.180	0.112	0.738	0.112	13.048	2.131
AVB	0.663	0.079	0.255	0.107	0.705	0.107	13.158	2.102
ESS	0.523	0.083	0.374	0.113	0.622	0.113	12.092	1.898
CPT	0.630	0.080	0.340	0.109	0.656	0.109	12.981	2.050
BRE	0.508	0.081	0.354	0.110	0.701	0.110	12.278	1.919
HME	0.380	0.081	0.377	0.110	0.545	0.111	10.603	1.643
AIV	0.719	0.095	0.488	0.129	0.892	0.130	15.339	2.515
MAA	0.527	0.079	0.243	0.107	0.377	0.107	10.487	1.667
PPS	0.688	0.088	0.266	0.119	0.652	0.120	13.150	2.160
CLP	0.390	0.076	0.445	0.103	0.553	0.103	10.970	1.662
AEC	0.442	0.111	0.592	0.151	0.711	0.151	12.665	2.179
BXP	0.616	0.079	0.158	0.107	0.536	0.107	11.637	1.882
SLG	0.769	0.098	0.539	0.133	0.776	0.133	15.378	2.542
ARE	0.389	0.079	0.405	0.107	0.580	0.107	10.946	1.680
DRE	0.694	0.087	0.333	0.118	0.753	0.118	13.916	2.257
CLI	0.592	0.084	0.144	0.114	0.664	0.114	11.994	1.963
KRC	0.653	0.094	0.618	0.128	0.688	0.128	14.325	2.332
HIW	0.638	0.094	0.333	0.128	0.824	0.128	13.810	2.272
OFC	0.491	0.090	0.535	0.123	0.680	0.123	12.701	2.034
BDN	0.710	0.098	0.539	0.133	0.918	0.133	15.576	2.563
PKY	0.672	0.104	0.677	0.141	0.781	0.141	15.133	2.516
SPG	0.508	0.088	0.314	0.119	0.708	0.119	12.164	1.950
MAC	0.575	0.101	0.548	0.137	0.799	0.137	13.981	2.306
KIM	0.590	0.093	0.276	0.126	0.604	0.126	12.181	2.006
FRT	0.413	0.078	0.308	0.106	0.632	0.106	11.023	1.695
O	0.319	0.073	0.386	0.099	0.560	0.099	10.222	1.534

Table 6 – Continued

Ticker	$\beta_{j,m}$	Std ($\beta_{j,m}$)	$\beta_{j,smb}$	Std ($\beta_{j,smb}$)	$\beta_{j,hml}$	Std ($\beta_{j,hml}$)	CoEC	Std (CoEC)
TCO	0.668	0.084	0.232	0.115	0.612	0.115	12.674	2.070
REG	0.448	0.084	0.354	0.113	0.532	0.114	10.996	1.723
DDR	0.761	0.107	0.165	0.145	0.760	0.145	13.869	2.445
WRI	0.472	0.084	0.241	0.115	0.639	0.115	11.284	1.797
SKT	0.428	0.082	0.135	0.111	0.453	0.111	9.660	1.549
CBL	0.641	0.106	0.847	0.144	0.934	0.144	16.236	2.722
EQY	0.556	0.079	0.328	0.107	0.517	0.107	11.695	1.829
ALX	0.698	0.112	0.476	0.152	0.508	0.152	13.318	2.324
GRT	0.848	0.121	0.561	0.165	0.906	0.165	16.699	2.899
AKR	0.453	0.077	0.205	0.105	0.446	0.105	10.081	1.567
BFS	0.303	0.098	0.410	0.134	0.585	0.134	10.301	1.742
PEI	0.729	0.110	0.601	0.149	0.936	0.149	16.046	2.702
RPT	0.514	0.105	0.362	0.143	0.578	0.143	11.768	2.000
OLP	0.468	0.071	0.308	0.097	0.488	0.097	10.775	1.617
ADC	0.381	0.094	0.489	0.128	0.596	0.128	11.260	1.841
RPI	0.295	0.114	0.237	0.155	0.393	0.155	8.697	1.672
SSS	0.368	0.089	0.358	0.121	0.533	0.121	10.381	1.667
Ave	0.568	0.093	0.369	0.126	0.666	0.126	12.638	2.082

Notes: Table 6 reports market risk premium loadings ($\beta_{j,m}$), SMB loading ($\beta_{j,smb}$) , HML loading ($\beta_{j,hml}$),and their associated uncertainties (Std ($\beta_{j,m}$), Std ($\beta_{j,smb}$), and Std ($\beta_{j,hml}$)). Moreover, we report cost of equity capital (*CoEC*) and uncertainty of cost of equity (*Std (CoEC)*). The model is Fama and French (1993) three-factor CoE:

$$R_{j,t} - R_{f,t-1} = \alpha_j + \beta_{j,m}(R_{m,t} - R_{f,t-1}) + \beta_{j,smb}R_{smb,t} + \beta_{j,hml}R_{hml,t}$$

estimated via full sample ordinary least squares (OLS). We report result of our 60-firm sample. The period is January 1999 to December 2011. CoECs and their associated uncertainties are in percent per year. As factors, we use the annual risk free rate and market risk premium data from 1927 to 2011. The average annual risk free rate is 3.63% while the average and standard error of the annual market risk premium is 7.94% and 2.26%. *Std (CoEC)* is estimated as the square root of

$$sqrt\left[RP^{'}Var\left(e\left(B\right)^{'}\right)RP + B^{'}Var\left(e\left(RP\right)\right)B + Var\left(e\left(B\right)^{'}\right)Var\left(e\left(RP\right)\right)\right]$$

where RP is replaced by long run mean of factors $RP = \left(\bar{R}_m - \bar{R}_f,\ \bar{R}_{smb},\ \bar{R}_{hml}\right)^{'}$, B is replaced by OLS estimates $\hat{B} = \left(\hat{\beta}_m,\ \hat{\beta}_{smb},\ \hat{\beta}_{hml}\right)^{'}$. Furthermore, $Var\left(e\left(RP\right)\right)$ corresponds to the squared standard error of the factors. Standard errors of the factors are calculated as the time series standard deviation of the factors divided by the square root of the number of data. $Var\left(e\left(B\right)^{'}\right)$ is replaced by the squared standard errors of the estimated OLS loadings. *Ave* is average of 60 firms.

Table 7 – Hierarchical Bayesian Fama and French (1993) three-factor cost of equity capital for our 60-firm sample.

Ticker	$\beta_{j,m}$	Std ($\beta_{j,m}$)	$\beta_{j,smb}$	Std ($\beta_{j,smb}$)	$\beta_{j,hml}$	Std ($\beta_{j,hml}$)	CoEC	Std (CoEC)
EPR	0.388	0.126	0.384	0.174	0.386	0.185	9.940	1.964
WRE	0.313	0.087	0.382	0.130	0.534	0.135	10.038	1.652
LXP	0.385	0.121	0.279	0.184	0.566	0.175	10.384	1.988
CUZ	0.429	0.134	0.240	0.185	0.418	0.194	9.893	2.042
IRETS	0.177	0.077	0.161	0.095	0.235	0.107	6.737	1.082
HCP	0.368	0.122	0.006	0.163	0.378	0.175	8.365	1.789
HCN	0.302	0.107	0.164	0.146	0.325	0.153	8.167	1.569
OHI	0.445	0.271	0.104	0.332	0.308	0.333	8.997	3.275
HR	0.477	0.136	0.174	0.172	0.511	0.175	10.474	2.078
LTC	0.343	0.143	0.099	0.194	0.292	0.219	8.095	1.998
PLD	0.627	0.120	-0.011	0.154	0.323	0.171	10.092	2.077
EGP	0.394	0.098	0.184	0.129	0.414	0.126	9.394	1.604
FR	0.555	0.172	0.176	0.217	0.595	0.215	11.491	2.522
MNRTA	0.202	0.072	0.133	0.101	0.243	0.108	6.864	1.089
HPT	0.484	0.135	0.078	0.178	0.351	0.180	9.416	2.010
LHO	0.777	0.183	0.236	0.247	0.496	0.268	13.002	2.967
FCH	1.151	0.359	0.284	0.455	0.908	0.473	18.102	5.137
EQR	0.551	0.121	0.096	0.162	0.534	0.159	10.884	2.065
AVB	0.560	0.110	0.147	0.154	0.509	0.155	11.021	2.008
ESS	0.429	0.107	0.231	0.144	0.429	0.155	9.912	1.786
CPT	0.533	0.107	0.223	0.141	0.466	0.164	10.882	1.956
BRE	0.427	0.106	0.212	0.133	0.491	0.150	10.120	1.786
HME	0.310	0.108	0.245	0.137	0.355	0.150	8.665	1.601
AIV	0.586	0.152	0.291	0.204	0.585	0.206	12.111	2.455
MAA	0.439	0.097	0.147	0.130	0.259	0.134	8.871	1.590
PPS	0.556	0.135	0.156	0.170	0.436	0.177	10.674	2.131
CLP	0.323	0.098	0.301	0.134	0.376	0.147	9.072	1.596
AEC	0.323	0.147	0.332	0.209	0.384	0.214	9.230	2.106
BXP	0.515	0.095	0.092	0.147	0.383	0.150	9.861	1.794
SLG	0.620	0.155	0.327	0.194	0.506	0.207	12.145	2.469
ARE	0.321	0.097	0.264	0.135	0.393	0.141	9.007	1.570
DRE	0.576	0.124	0.199	0.156	0.524	0.181	11.409	2.158
CLI	0.484	0.108	0.060	0.143	0.463	0.160	9.885	1.859
KRC	0.530	0.137	0.399	0.197	0.432	0.205	11.339	2.275
HIW	0.498	0.135	0.186	0.183	0.547	0.194	10.852	2.175
OFC	0.394	0.125	0.347	0.177	0.438	0.169	10.097	1.943
BDN	0.574	0.150	0.321	0.206	0.593	0.226	12.165	2.490
PKY	0.524	0.164	0.414	0.224	0.458	0.238	11.465	2.511
SPG	0.412	0.119	0.175	0.151	0.471	0.162	9.770	1.851
MAC	0.451	0.154	0.324	0.198	0.500	0.231	10.761	2.325
KIM	0.455	0.127	0.148	0.164	0.388	0.177	9.620	1.935
FRT	0.340	0.102	0.186	0.131	0.443	0.145	9.107	1.619
O	0.270	0.089	0.272	0.126	0.403	0.140	8.670	1.480

Table 7 – Continued

Ticker	$\beta_{j,m}$	Std ($\beta_{j,m}$)	$\beta_{j,smb}$	Std ($\beta_{j,smb}$)	$\beta_{j,hml}$	Std ($\beta_{j,hml}$)	CoEC	Std (CoEC)
TCO	0.557	0.114	0.124	0.144	0.426	0.162	10.521	1.974
REG	0.363	0.101	0.233	0.146	0.361	0.154	9.070	1.650
DDR	0.574	0.152	0.043	0.198	0.467	0.219	10.551	2.358
WRI	0.370	0.103	0.124	0.149	0.436	0.155	9.080	1.685
SKT	0.339	0.100	0.061	0.130	0.300	0.136	7.963	1.486
CBL	0.497	0.159	0.503	0.234	0.557	0.247	12.046	2.583
EQY	0.471	0.100	0.209	0.137	0.358	0.136	9.831	1.724
ALX	0.512	0.164	0.275	0.217	0.277	0.218	10.008	2.333
GRT	0.621	0.193	0.290	0.264	0.511	0.273	12.041	2.873
AKR	0.372	0.099	0.110	0.116	0.312	0.132	8.457	1.507
BFS	0.225	0.124	0.222	0.155	0.336	0.180	7.819	1.691
PEI	0.561	0.185	0.341	0.237	0.566	0.264	12.003	2.747
RPT	0.387	0.139	0.169	0.195	0.330	0.202	8.882	2.000
OLP	0.395	0.091	0.217	0.129	0.363	0.133	9.274	1.573
ADC	0.295	0.124	0.294	0.155	0.365	0.170	8.778	1.761
RPI	0.188	0.134	0.101	0.171	0.179	0.187	6.336	1.681
SSS	0.286	0.109	0.219	0.146	0.327	0.165	8.253	1.608
Ave	0.447	0.130	0.212	0.174	0.425	0.184	9.966	2.027

Notes: Table 7 reports market risk premium loadings ($\beta_{j,m}$), SMB loading ($\beta_{j,smb}$) , HML loading ($\beta_{j,hml}$),and their associated uncertainties (Std ($\beta_{j,m}$), Std ($\beta_{j,smb}$), and Std ($\beta_{j,hml}$)). Furthermore, we present cost of equity capital A(*CoEC*) and uncertainty of cost of equity (*Std (CoEC)*). The model is Fama and French (1993) three-factor CoE estimated within a Bayesian Hierarchical framework. We report result of our 60-firm sample. The period is January 1999 to December 2011. CoECs and their associated uncertainties are in percent per year. As factors, we use the annual risk free rate and market risk premium data from 1927 to 2011. We generate 200 loadings and CoEC estimates via Markov Chain Montel Carlo (MCMC) framework. The average annual risk free rate is 3.63% while the average and standard error of the annual market risk premium is 7.94% and 2.26%. *Std (CoEC)* is estimated as the square root of

$$sqrt\left[RP'Var\left(e\left(B\right)'\right)RP + B'Var\left(e\left(RP\right)\right)B + Var\left(e\left(B\right)'\right)Var\left(e\left(RP\right)\right)\right]$$

where RP is replaced by long run mean of factors $RP = \left(\bar{R}_m - \bar{R}_f,\ \bar{R}_{smb},\ \bar{R}_{hml}\right)'$, B is replaced by average Bayes estimates $\hat{B} = \left(\hat{\beta}_m,\ \hat{\beta}_{smb},\ \hat{\beta}_{hml}\right)'$. Furthermore, $Var\left(e\left(RP\right)\right)$ corresponds to the squared standard error of the factors. Standard errors of the factors are calculated as the time series standard deviation of the factors divided by the square root of the number of data. $Var\left(e\left(B\right)'\right)$ is replaced by the variance of the estimated loadings. *Ave* is average of 60 firms.